Structure of the Book

In this book many topics are discussed at a fairly advanced level but using a pedagogical approach. I believe that a student could highly profit from some exposure to such treatments.

The book is divided in four parts.

Part I: **Preliminary notions** (Chapters 1, 2, and 3)

The first part deals with the fundamental aspects of phase transitions, illustrated by explicit examples coming from the Ising model or similar systems.

Chapter 1: a straighforward introduction of essential ideas on second-order phase transitions and their theoretical challenge. Our attention focuses on some important issues, such as order parameters, correlation length, correlation functions, scaling behavior, critical exponents, etc. A short discussion is also devoted to the Ising model and its most significant developments during the years of its study. The chapter also contains two appendices, where all relevant results of classical and statistical mechanics are summarized.

Chapter 2: this deals with one-dimensional statistical models, such as the Ising model and its generalizations (Potts model, systems with $O(n)$ or Z_n symmetry, etc.). Several methods of solution are discussed: the recursive method, the transfer matrix approach or series expansion techniques. General properties of these methods – valid on higher dimensional lattices – are also highlighted. The contents of this chapter are quite simple and pedagogical but extremely useful for understanding the rest of the book. One of the appendices at the end of the chapter is devoted to a famous problem of topology, i.e. the four-color problem, and its relation with the two-dimensional Potts model.

Chapter 3: here we discuss the approximation schemes to approach lattice statistical models that are not exactly solvable. In addition to the mean field approximation, we also consider the Bethe–Peierls approach to the Ising model. Moreover, there is a thorough discussion of the gaussian model and its spherical version – two important systems with several points of interest. In one of the appendices there is a detailed analysis of the random walk on different lattices: apart from the importance of the subject on its own, it is shown that the random walk is responsible for the critical properties of the spherical model.

Part II: **Two-dimensional lattice models** (Chapters 4, 5, and 6)

This part provides a general introduction to the key ideas of equilibrium statistical mechanics of discrete systems.

Chapter 4: at the beginning of this chapter there is the Peierls argument (it permits us to prove the existence of a phase transition in the two-dimensional Ising model). The rest of the chapter deals with the the duality transformations that link the low- and the high-temperature phases of several statistical models. Particularly important is the proof of the so-called *star–triangle identity.* This identity will be crucial in the later discussion of the transfer matrix of the Ising model (Chapter 6).

Chapter 5: two exact combinatorial solutions of the two-dimensional Ising model are the key topics of this chapter. Although no subsequent topic depends on them, both the mathematical and the physical aspects of these solutions are elegant enough to deserve special attention.

Chapter 6: this deals with the exact solution of the two-dimensional Ising model achieved through the transfer matrix formalism. A crucial role is played by the commutativity properties of the transfer matrices, which lead to a functional equation for their eigenvalues. The exact free energy of the model and its critical point can be identified by means of the lowest eigenvalue. We also discuss the general structure of the Yang–Baxter equation, using the six-vertex model as a representative example.

Part III: **Quantum field theory and conformal invariance** (Chapters 7–14)

This is the central part of the book, where the aims of quantum field theory and some of its fundamental results are discussed. A central point is the bootstrap method of conformal field theories. The main goal of this part is to show the extraordinary efficiency of these techniques for the analysis of critical phenomena.

Chapter 7: the main reasons for adopting the methods of quantum field theory to study the critical phenomena are emphasized here. Both the canonical quantization and the path integral formulation of the field theories are presented, together with the analysis of the perturbation theory. Everything in this chapter will be needed sooner or later, since it highlights most of the relevant aspects of quantum field theory.

Chapter 8: the key ideas of the renormalization group are introduced here. They involve the scaling transformations of a system and their implementations in the space of the coupling constants. From this analysis, one gets to the important notion of relevant, irrelevant, and marginal operators and then to the universality of the critical phenomena.

Chapter 9: a crucial aspect of the Ising model is its fermionic nature and this chapter is devoted to this property of the model. In the continuum limit, a Dirac equation for neutral Majorana fermions emerges. The details of the derivation are much less important than understanding why it is possible. The simplicity and the exactness of the result are emphasized.

Chapter 10: this chapter introduces the notion of conformal transformations and the important topic of the massless quantum field theories associated to the critical points of the statistical models. Here we establish the important conceptual result that the classification of all possible critical phenomena in two dimensions consists of finding out all possible irreducible representations of the Virasoro algebra.

Chapter 11: the so-called minimal conformal models, characterized by a finite number of representations, are discussed here. It is shown that all correlation functions of these models satisfy linear differential equations and their explicit solutions are given by using the Coulomb gas method. Their exact partition functions can be obtained by enforcing the modular invariance of the theory.

Chapter 12: free theories are usually regarded as trivial examples of quantum systems. This chapter proves that this is not the case of the conformal field theories associated to the free bosonic and fermionic fields. The subject is not only full of beautiful mathematical identities but is also the source of deep physical concepts with far reaching applications.

Chapter 13: the conformal transformations may be part of a larger group of symmetry and this chapter discusses several of their extensions: supersymmetry, Z_n transformations, and current algebras. In the appendix the reader can find a self-contained discussion on Lie algebras.

Chapter 14: the identification of a class of universality is one of the central questions in statistical physics. Here we discuss in detail the class of universality of several models, such as the Ising model, the tricritical Ising model, and the Potts model.

Part IV: **Away from criticality** (Chapters 15–21)

This part of the book develops the analysis of the statistical models away from criticality.

Chapter 15: here is introduced the notion of *the scaling region* near the critical points, identified by the deformations of the critical action by means of the relevant operators. The renormalization group flows that originate from these deformations are subjected to important constraints, which can be expressed in terms of sum rules. This chapter also discusses the nature of the perturbative series based on the conformal theories.

Chapter 16: the general properties of the integrable quantum field theories are the subject of this chapter. They are illustrated by means of significant examples, such as the Sine–Gordon model or the Toda field theories based on the simple roots of a Lie algebra. For the deformations of a conformal theory, it is shown how to set up an efficient counting algorithm to prove the integrability of the corresponding model.

Chapter 17: this deals with the analytic theory of the S-matrix of the integrable models. Particular emphasis is put on the dynamical principle of the *bootstrap*, which gives rise to a recursive structure of the amplitudes. Several dynamical quantities, such as mass ratios or three-coupling constants, have an elegant mathematic formulation, which also has an easy geometrical interpretation.

Chapter 18: the Ising model in a magnetic field is one of the most beautiful example of an integrable model. In this chapter we present its exact S-matrix and the exact spectrum of its excitations, which consist of eight particles of different masses. Similarly, we discuss the exact scattering theory behind the thermal deformation of the tricritical Ising model and the unusual features of the exact S-matrix of the non-unitary Yang–Lee model. Other important examples are provided by $O(n)$ invariant models: when $n = 2$, one obtains the important case of the Sine–Gordon model. We also discuss the quantum-group symmetry of the Sine–Gordon model and its reductions.

Chapter 19: the thermodynamic Bethe ansatz permits us to study finite size and finite temperature effects of an integrable model. Here we derive the integral equations that determine the free energy and we give their physical interpretation.

Chapter 20: at the heart of a quantum field theory are the correlation functions of the various fields. In the case of integrable models, the correlators can be expressed in terms of the spectral series based on the matrix elements on the asymptotic states. These matrix elements, also known as form factors, satisfy a set of functional and recursive equations that can be exactly solved in many cases of physical interest.

Chapter 21: this chapter introduces a perturbative technique based on the form factors to study non-integrable models. Such a technique permits the computation of the corrections to the mass spectrum, the vacuum energy, the scattering amplitudes, and so on.

Problems Each chapter of this book includes a series of problems. They have different levels of difficulty: some of them relate directly to the essential material of the chapters, other are instead designed to introduce new applications or even new topics. The problems are an integral part of the course and their solution is a crucial step for the understanding of the whole subject.

Mathematical aspects Several chapters have one or more appendices devoted to some mathematical aspects encountered in the text. Far from being a collection of formulas, these appendices aim to show the profound relationship that links mathematics and physics. Quite often, they also give the opportunity to achieve comprehension of mathematical results by means of physical intuition. Some appendices are also devoted to put certain ideas in their historical perspective in one way or another.

References At the end of each chapter there is an annotated bibliography. The list of references, either books or articles, is by no means meant to be a comprehensive