Cosmological Neutrinos

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Varenna – ISAPP23 PhD School – 02 and 03/07/23

Plan

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

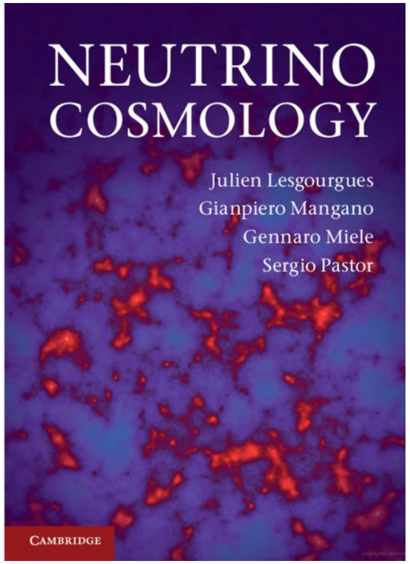
Some references

Lesgourgues and Pastor 2006 review on the arXiv

Wong https://arxiv.org/pdf/1111.1436.pdf

Lesgourgues, Mangano, Miele, Pastor “Neutrino Cosmology” 2013 Cambridge University Press

Ma & Bertschinger https://arxiv.org/pdf/astro-ph/9506072.pdf



Drop me an email if you wish further references: viel AT sissa.it

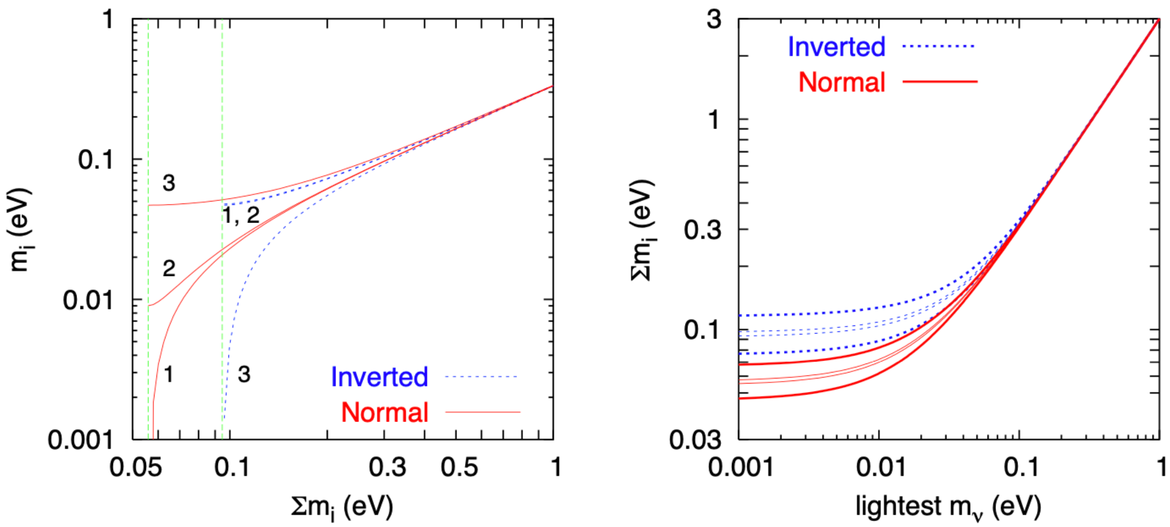
Linear Perturbation theory

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Facts

Boundary conditions from **particle physics**

$$0.056\,(0.095)\ \mathrm{eV} \lesssim \sum_i m_i \lesssim 6\ \mathrm{eV}$$



We will see that cosmology will be sensitive to total neutrino mass

The neutrino background: neutrino decoupling

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Alpher, Follin, Herman 1953
CνB

Firm established prediction of the standard Big Bang model

$$f_{\rm eq}(p) = \left[\exp\left(\frac{p-\mu_\nu}{T}\right)+1\right]^{-1}$$

With chemical potential ~ 0 BBN sets tight limits

$$n_\nu = \frac{g}{(2\pi)^3}\int d^3p\; f_\nu(E,T_\nu), \qquad \rho_\nu = \frac{g}{(2\pi)^3}\int d^3p\; E\; f_\nu(E,T_\nu),$$

Weak interaction rate

$$\Gamma_\nu = \langle \sigma_\nu \, n_\nu \rangle$$

Hubble parameter

$$H = \sqrt{\frac{8\pi\rho}{3M_P^2}}$$

T_{dec} ~ MeV [at 1 sec]

After decoupling: f_{eq} is preserved because T and p scale as 1/a
No dependence on the mass

This means that momentum distribution is exact even in the epochs of structure formation!

The neutrino background: energy densities

After neutrino decoupling photon temperature drops below electron mass, e+e- annihilation heat the plasma [we are at T~0.5 MeV or so]

$$n_\nu = \frac{3}{11} n_\gamma = \frac{6\zeta(3)}{11\pi^2} T_\gamma^3$$

n_ν / n_γ = 0.68
At any time after electron
Positron annihilation

$$\rho_\nu(m_\nu \ll T_\nu) = \frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{4/3} T_\gamma^4$$
$$\rho_\nu(m_\nu \gg T_\nu) = m_\nu n_\nu \ .$$

Two well-defined limits for matter and radiation

Note: there are small non-thermal distortions in the neutrino FD spectrum and a slight increase in the photon neutrino temperature due to relic interactions of e+ e- with neutrinos [e.g. Dolgov 02]

The neutrino background: today

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Extrapolating to today:

$n_\nu = 339.5\ \mathrm{cm}^{-3}$ average

Note: clustering in the local Universe can slightly change this number

$T_\gamma = 2.73\ \mathrm{K} \rightarrow T_\nu = 1.95\ \mathrm{K}$

$\langle p \rangle = 1.7 \times 10^{-4}\ \mathrm{eV} = 3.15\ T_\nu$

$$z_{\rm nr} = \left(\frac{m_\nu}{5.28 \times 10^{-4}\mathrm{eV}}\right)\left(\frac{T_\nu^a}{T_\nu}\right) - 1$$

$$\Omega_\nu h^2 = \frac{m_\nu}{93.14\ \mathrm{eV}}$$

$\Omega_\nu h^2 \geq 6 \times 10^{-4}$ (NO), or $\geq 10^{-3}$ (IO)

$\Omega_\nu > 0.5\%$ of matter components – sub-dominant matter component
Hot DM
5th most abundant Universe component by energy
2nd most abundant by number density

Very small numbers → direct detection difficult

The neutrino background – radiation era

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$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_\gamma$$

Extra radiation contribution

Neff= energy density of neutrinos + other light particles / energy density of 1neutrino family neglecting e+e-> ν

Neff is accurately theoretically estimated to be 3.044 and also measured from cosmological observations

$$N_{\rm eff} = 3.00^{+0.57}_{-0.53} \quad (95\,\%, \textit{Planck}\ \text{TT+lowE}),$$

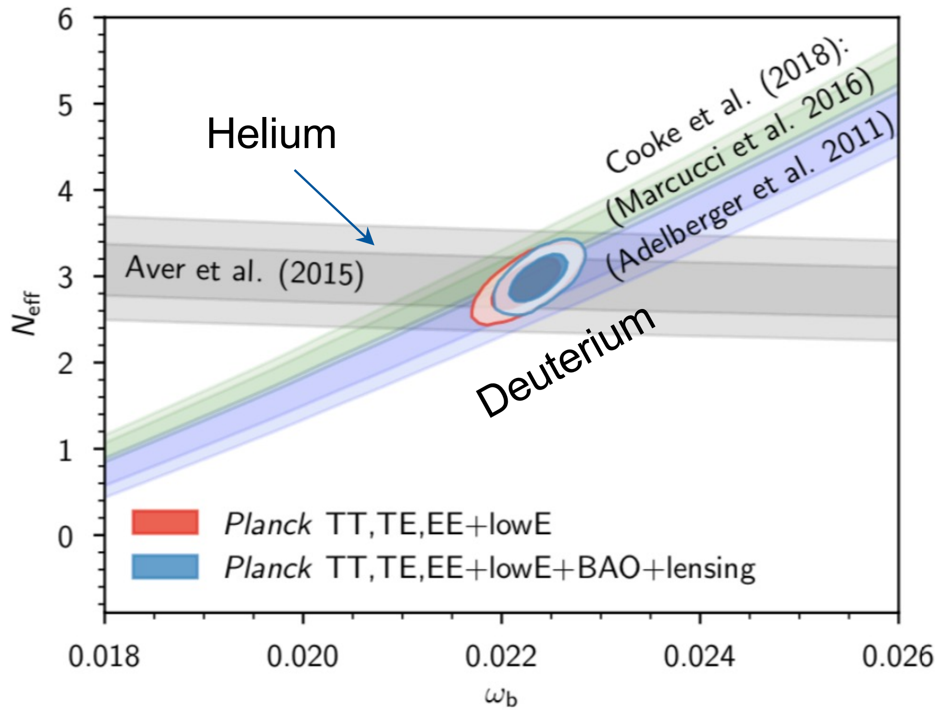
$$N_{\rm eff} = 2.92^{+0.36}_{-0.37} \quad (95\,\%, \textit{Planck}\ \text{TT,TE,EE+lowE}),$$

From CMB Planck 2018

Also BBN provides constraints and the error bar can be reduced by a factor 2 see Yeh+22

Convincing proof of existence of cosmic neutrino background

N_{eff}



Planck collaboration VI, 2018

Douglas Scott's lectures

Remarkable success of cosmological data in probing particle dark matter (including Non-standard heat injection in the primordial Universe)

Neff now detected at >10-20σ

But there is more:
Detected anisotropies/perturbations in the fluid and their imprint on the CMB (especially polarization)

Everything consistent with a Relativistic fluid with no viscosity!

N_{eff} to test particle (new) physics

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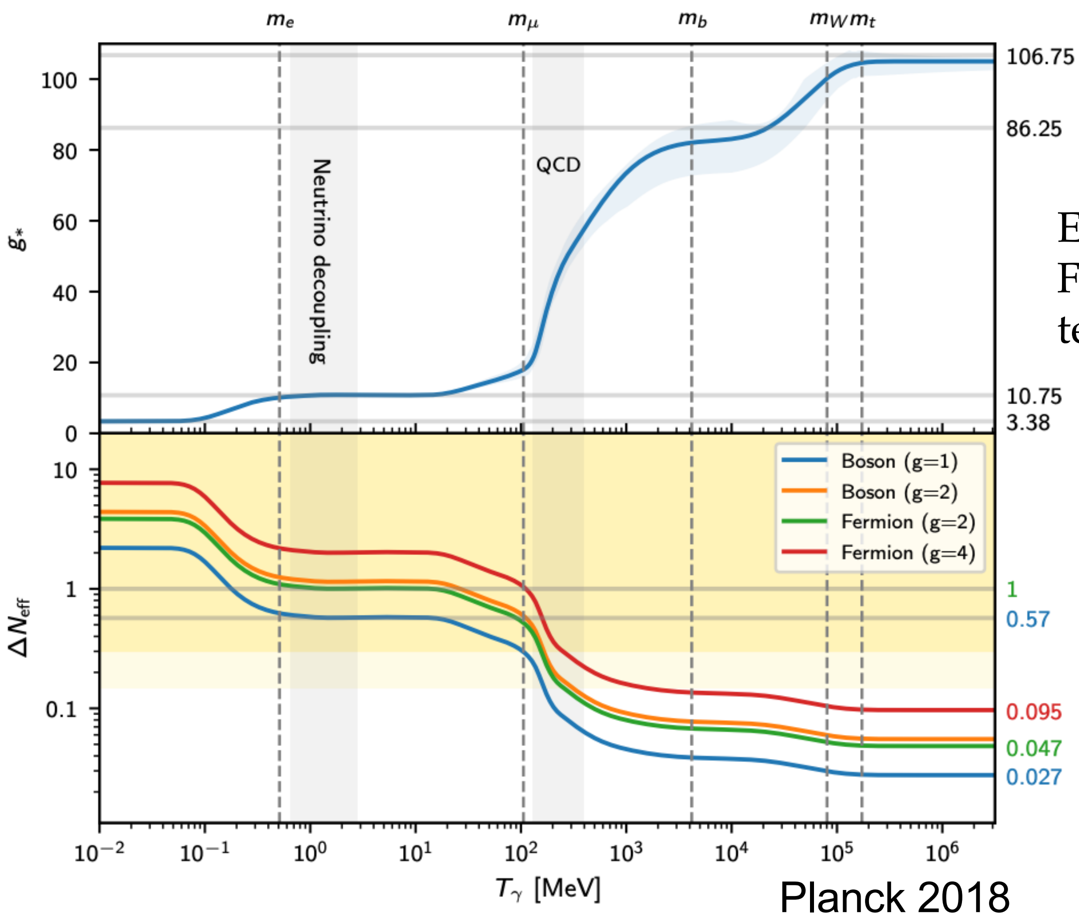
Constraints on additional relativistic particles

$$\Delta N_{\rm eff} = g\left[\frac{43}{4\, g_s}\right]^{4/3} \times \begin{cases} 4/7 & \text{boson,} \\ 1/2 & \text{fermion,} \end{cases}$$

Fully thermalized relics

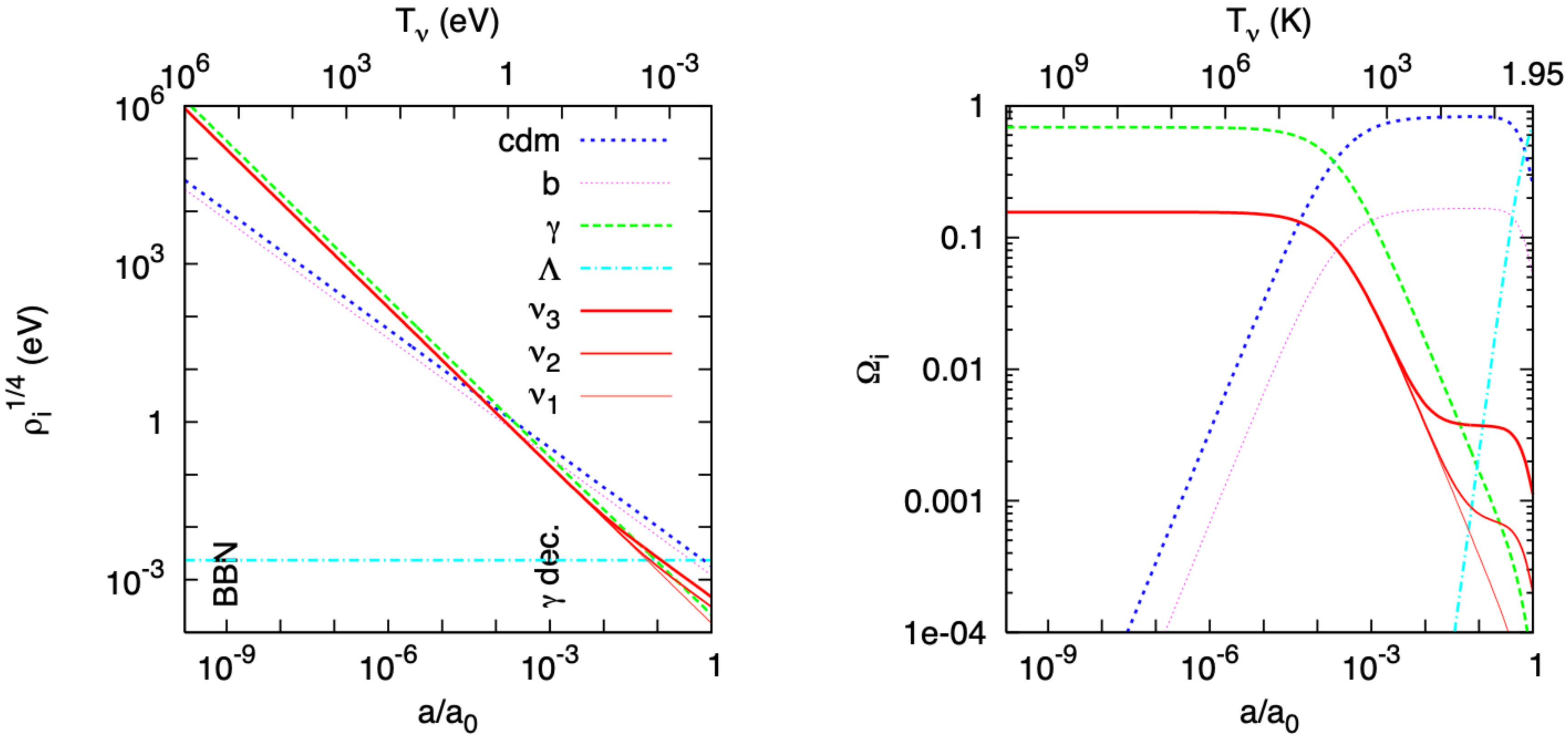


Planck 2018

Evolution of effective degrees of Freedom for SM particles vs photon temperature

Expected ΔN_{eff} today
For species decoupling
From thermal equilibrium
At $T\gamma$

Evolution of energy densities in a neutrino Universe



Normal hierarchy with m_1=0 eV, m_2=0.009 eV, m_3= 0.05eV

The perturbed Universe

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1. The metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(\tau)[(1+2\phi)d\tau^2 - (1-2\psi)\delta_{ij}dx^i dx^j]$$

Conformal Newtonian longitudinal gauge and look only into scalar perturbations → LSS

2. The tensor

$$\delta T^0_0 = \delta\rho \; ,$$

Energy density perturbation

$$\delta T^0_i = (\bar\rho + \bar p) v_i^{||} \; ,$$

$v_i^{||}$ Longitudinal component of velocity field

$$\delta T^i_j = -\delta p \, \delta^i_j + \Sigma^{i||}_j \; ,$$

δp pressure perturbation, traceless and longitudinal component of the 3x3 tensor

$$\Sigma^{i||}_j = (\partial_i\partial_j - \tfrac{1}{3}\delta_{ij}\nabla^2)\tilde\sigma$$

3. New Variables

Velocity divergence

$$\theta \equiv \sum_i \partial_i v_i = \nabla^2 \tilde v \; ,$$

Shear (anisotropic) stress

$$(\bar\rho + \bar p)\nabla^2\sigma \equiv -\sum_{i,j}(\partial_i\partial_j - \frac{1}{3}\nabla^2\delta_{ij})\Sigma^{i||}_j = -\frac{2}{3}\nabla^4\tilde\sigma$$

The perturbed Universe - II

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4. Perturbed Einstein equation for the scalar sector (which imply conservation of total energy momentum-tensor)

$$\delta G^0_0 = 2a^{-2}\left\{-3\left(\frac{\dot a}{a}\right)^2\phi - 3\frac{\dot a}{a}\dot\psi + \nabla^2\psi\right\} = 8\pi G\ \delta\rho\ ,$$

$$\delta G^0_i = 2a^{-2}\partial_i\left\{\frac{\dot a}{a}\phi + \dot\psi\right\} = 8\pi G\ (\bar\rho + \bar p)\ v_i\ ,$$

$$\delta G^i_j = -2a^{-2}\left\{\left[\left(2\frac{\ddot a}{a} - \left(\frac{\dot a}{a}\right)^2\right)\phi + \frac{\dot a}{a}(\dot\phi + 2\dot\psi) + \ddot\psi + \frac{1}{3}\nabla^2(\phi - \psi)\right]\delta^i_j\right.$$
$$\left. -\frac{1}{2}\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i_j\right)(\phi - \psi)\right\} = 8\pi G\ (-\delta p\ \delta^i_j + \Sigma^i_j)$$

5. Perturbed Einstein equations + change of variable + let's move to Fourier space

$$-3\left(\frac{\dot a}{a}\right)^2\phi - 3\frac{\dot a}{a}\dot\psi - k^2\psi = 4\pi G\ a^2\ \bar\rho\ \delta\ ,$$

$$-k^2\left(\frac{\dot a}{a}\phi + \dot\psi\right) = 4\pi G\ a^2\ (\bar\rho + \bar p)\ \theta\ ,$$

$$\left(2\frac{\ddot a}{a} - \left(\frac{\dot a}{a}\right)^2\right)\phi + \frac{\dot a}{a}(\dot\phi + 2\dot\psi) + \ddot\psi - \frac{k^2}{3}(\phi - \psi) = 4\pi G\ a^2\ \delta p\ ,$$

$$k^2(\phi - \psi) = 12\pi G\ a^2\ (\bar\rho + \bar p)\ \sigma$$

The perturbed Universe - III

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6. ….but we are dealing with fluids: continuity for each fluid component

$$\dot{\delta} = (1+w)(\theta + 3\dot{\psi})$$

7. … and Euler for **each** fluid component

$$\dot{\theta} = \frac{\dot{a}}{a}(3w-1)\theta - \frac{\dot{w}}{1+w}\theta - k^2\phi - k^2\sigma - \frac{w}{1+w}k^2\delta$$

$w = \bar{p}/\bar{\rho} = \delta p/\delta\rho.$

8. … and a perfect fluid → energy-momentum tensor is diagonal and isotropic

$$T^{\mu\nu} = -p\ g^{\mu\nu} + (\rho + p)U^\mu U^\nu$$

$U^\mu = dx^\mu/[a(1+\phi)d\tau]$ and obtain:

$$U^\mu = \left(a^{-1}[1-\phi],\, a^{-1}v^i\right)\ , \qquad T^0_0 = \rho\ , \quad T^i_0 = v^i\ , \quad T^i_i = -p\ ,$$

The perturbed Universe - IV

9. Now solve Einstein equations in a **neutrinoless** Univers with the perturbed energy-momentum tensor

$$\delta T^0_0 = \delta\rho_{\rm r} + \delta\rho_{\rm m} \ ,$$

$$\partial^i(\delta T^0_i) = (\bar\rho_{\rm r} + \bar p_{\rm r})\theta_{\rm r} + \bar\rho_{\rm m}\theta_{\rm m} = \frac{4}{3}\bar\rho_{\rm r}\theta_{\rm r} + \bar\rho_{\rm m}\theta_{\rm m} \ ;$$

$$\delta T^i_i = -\delta p_{\rm r} = -\frac{1}{3}\delta\rho_{\rm r}$$

At this point it is very important to define a **Jeans length**

Causal Horizon/Particle Horizon
(maximum physical
scale at which a signal can propagate)

$$d(t_i, t) = a(t)\int_{t_i}^{t} dx = a(t)\int_{t_i}^{t}\frac{v\,dt'}{a(t')}$$

Hubble radius/Particle Horizon
(for a ~ t^n and n<1)

$$R_H(t) = \frac{t}{n}\ , \qquad d_H(t \gg t_i) \simeq \frac{t}{1-n}\ .$$

Acoustic perturbations c_s
Sound Horizon $c_s/H(t)$

$$k_J(t) = \left(\frac{4\pi G\bar\rho(t)a^2(t)}{c_s^2(t)}\right)^{1/2}\ , \qquad \lambda_J(t) = 2\pi\frac{a(t)}{k_J(t)} = 2\pi\sqrt{\frac{2}{3}}\frac{c_s(t)}{H(t)}$$

The perturbed Universe - V

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10. Evolution of perturbation in a fluid with perturbations propogating with sound speed cs

Two regimes

Large scales $k<k_J$ pressure unimportant (Jeans unstable)

$$\ddot{\delta} + \frac{\dot{a}}{a}\,\dot{\delta} + (k^2 - k_J^2)\,c_s^2\,\delta = 0$$

Small scales Modes with $k>k_J$ will oscillate
With frequency k x c_s (competition
between pressure and gravity) → they are Jeans stable

11. Jeans instability is a key ingredient for structure formation
 before recombination c_s~c the photon-baryon fluid oscillates on scales smaller than λ_J
 after recombination c_s→0 λ_J→0 and structure can grow

More in Enzo Branchini's lectures

The perturbed Universe - VI

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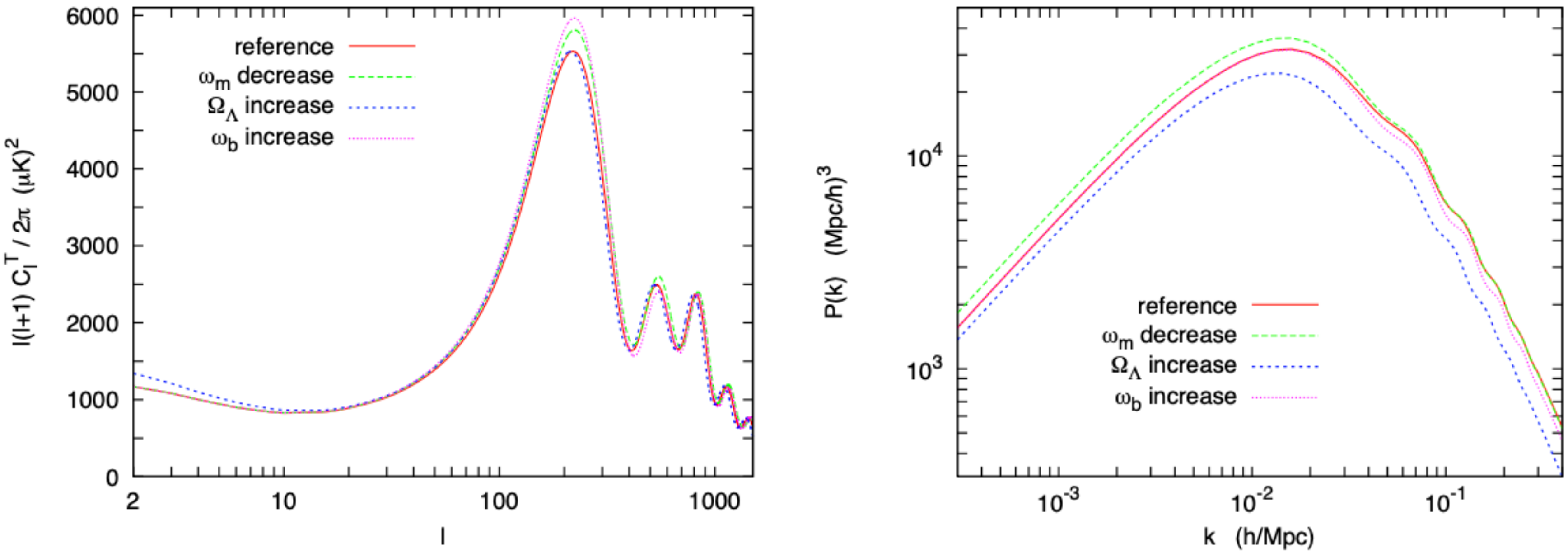
In a neutrinoless Universe



More in Douglas Scott's lectures

The perturbed Universe: adding neutrinos

At least 2 neutrinos should be matter – an extra matter component, this implies that they turned non-relativistic during matter domination z<3000

$$q_i \equiv a p_i,$$

$$P_i = a(1-\psi)p_i$$

Canonical conjugate
of the comoving coordinate x^i

$$f(x^i, P_j, \tau) = f_0(P, \tau)$$

In absence of perturbation pi will decade as 1/a while P_i will stay constant

$$\epsilon = a(p^2 + m^2)^{1/2} = (q^2 + a^2 m^2)^{1/2}$$

$$T^0_0 = \bar{\rho}_\nu = \frac{4\pi}{a^4} \int q^2 dq\, \epsilon\, f_0(q) \ ,$$

$$T^i_i = -\bar{p}_\nu = -\frac{4\pi}{3a^4} \int q^2 dq\, \frac{q^2}{\epsilon}\, f_0(q)$$

$$f_0(q) = \frac{1}{e^{q/aT_\nu} + 1}$$

BUT

Spatial perturbations in the metric will induce variations in neutrino phase-space density
Depending on time, space and momentum → this will impact on the energy momentum tensor

Scalar sector of the tensor will contain now the anisotropic stress (different w.r.t. perfect fluid)

The perturbed Universe: the energy momentum tensor

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$$f(x^i, P_j, \tau) = f_0(q)[1 + \Psi(x^i, q_j, \tau)] \qquad \text{with} \qquad P_j = (1 - \Psi)q_j \qquad \Psi \ll 1$$

$$T_{\mu\nu} = \int dP_1 dP_2 dP_3 \, (-g)^{-1/2} \, \frac{P_\mu P_\nu}{P^0} f(x^i, P_j, \tau)$$

Collisionless fluid with no
Microscopic interactions and
No bulk motions

$$T^0_0(x^i) = a^{-4} \int q^2 dq \, d\Omega \, \epsilon \, f_0(q) \, [1 + \Psi(x^i, q\hat{n}_j, \tau)] \, ,$$

$$T^0_i(x^i) = a^{-4} \int q^2 dq \, d\Omega \, q \, \hat{n}_i \, f_0(q) \, \Psi(x^i, q\hat{n}_j, \tau) \, ,$$

$d\Omega$ is the differential of the momentum direction $\hat{n}_j = q_j/q$

$$T^i_j(x^i) = -a^{-4} \int q^2 dq d\Omega \, \frac{q^2}{\epsilon} \, \hat{n}_i \hat{n}_j \, f_0(q) \, [1 + \Psi(x^i, q\hat{n}_j, \tau)] \, ,$$

$$(-g)^{-1/2} = a^{-4}(1 - \phi + 3\psi)$$

These are now the perturbed components

$$\delta\rho_\nu = a^{-4} \int q^2 dq \, d\Omega \, \epsilon f_0 \Psi \, , \qquad \delta P_\nu = \frac{1}{3} a^{-4} \int q^2 dq \, d\Omega \, \frac{q^2}{\epsilon} f_0 \Psi \, , \qquad (92)$$

$$\delta T^0_{i\,\nu} = a^{-4} \int q^2 dq \, d\Omega \, q\hat{n}_i \, f_0 \Psi \, , \quad \Sigma^i_{j\,\nu} = -a^{-4} \int q^2 dq \, d\Omega \, \frac{q^2}{\epsilon} (\hat{n}_i \hat{n}_j - \frac{1}{3}\delta_{ij}) \, f_0 \Psi \, .$$

The perturbed Universe: neutrino free streaming

As we defined the Jeans length we can now replace c_s with $_{\text{vthermal}}$

$$k_{FS}(t) = \left(\frac{4\pi G\bar{\rho}(t)a^2(t)}{v_{\rm th}^2(t)}\right)^{1/2}, \qquad \lambda_{FS}(t) = 2\pi\frac{a(t)}{k_{FS}(t)} = 2\pi\sqrt{\frac{2}{3}}\frac{v_{\rm th}(t)}{H(t)}$$

$$v_{\rm th} \equiv \frac{\langle p\rangle}{m} \simeq \frac{3T_\nu}{m} = \frac{3T_\nu^0}{m}\left(\frac{a_0}{a}\right) \simeq 150(1+z)\left(\frac{1\,\mathrm{eV}}{m}\right)\mathrm{km\,s^{-1}}$$

$$\lambda_{FS}(t) = 7.7\frac{1+z}{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}}\left(\frac{1\,\mathrm{eV}}{m}\right)h^{-1}\mathrm{Mpc}\ ,$$

$$k_{FS}(t) = 0.82\frac{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}}{(1+z)^2}\left(\frac{m}{1\,\mathrm{eV}}\right)h\,\mathrm{Mpc}^{-1},$$

The perturbed Universe: neutrino free streaming - II

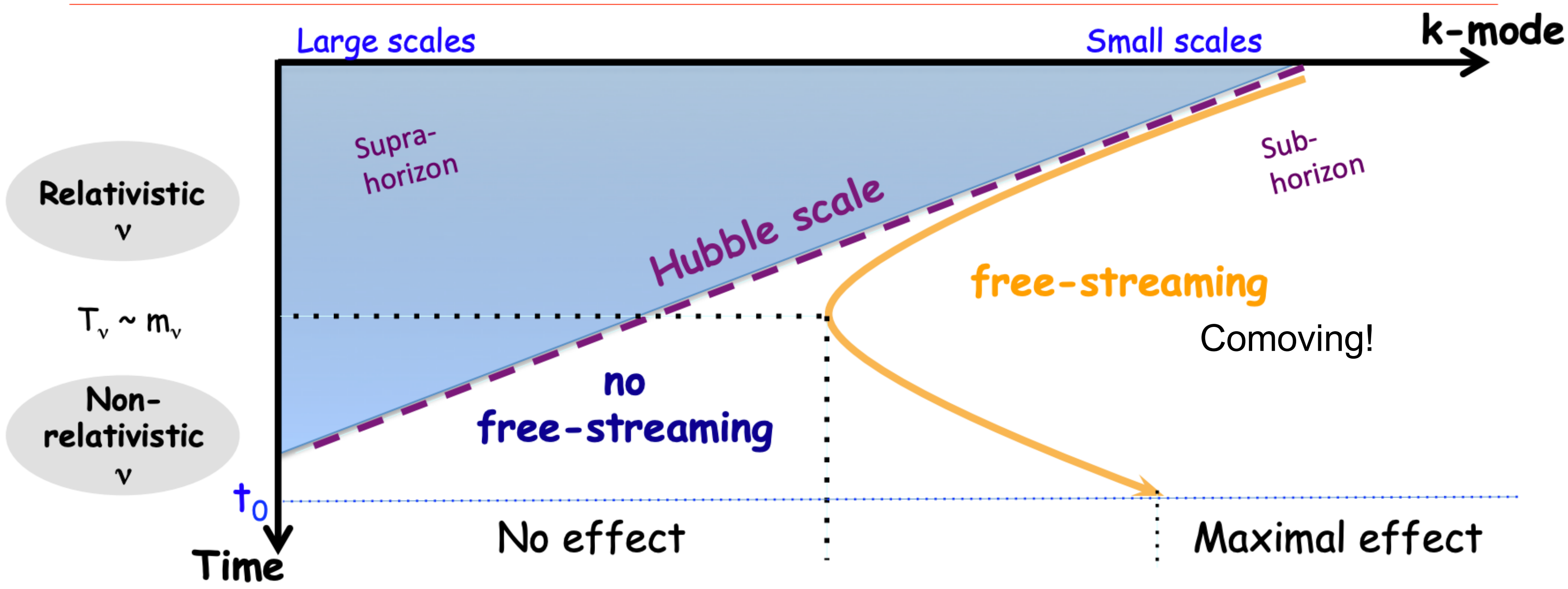
These scales are physical (not comoving) and set by particle physics and have to be compared to cosmic expansion
Before non-relativistic transition $\lambda_{FS} \sim t$ and $a \sim t^{1/2}$
After non-relativistic transition free-streaming scale increases $\lambda_{FS} \sim 1/(aH) \sim t^{1/3}$
But comoving $\lambda_{FS}/a \sim t^{-1/3}$, because $a \sim t^{2/3}$, comoving free streaming scale decreases!

Thus, if neutrinos become relativistic during MD the comoving free-streaming scale passes through a minimum at $k=k_{nr}$ when $m=\langle p\rangle=3T\nu=2000\ (m/1eV)$

$$k_{\rm nr} \simeq 0.018\ \Omega_{\rm m}^{1/2} \left(\frac{m}{1\,{\rm eV}}\right)^{1/2} h\,{\rm Mpc}^{-1}$$

Damping of perturbations below this scale
1- Neutrinos cannot be confined into these smaller scales
2- Metric perturbations will be damped at these scales by gravitational back-reaction
3- Modes at k<knr will evolve like in a neutrinoless universe

The perturbed Universe: neutrino free streaming - III



Taken from Palanque-Delabrouille PONT17 talk

The perturbed Universe: neutrino free streaming - IV

After decoupling neutrino free-stream

Free streaming scale is a dynamical quantity which quantifies which scales free-streaming can be neglected in the evolution equations at any given time

$$\lambda_{\rm FS} \equiv 2\pi \sqrt{\frac{2}{3}} \frac{c_\nu(\eta)}{H(\eta)}$$

Free streaming horizon is the average distance travelled by neutrinos between the early universe and a given time, displaying the scales that can by affected at allç

$$d_{\rm FS} = a r_{\rm FS} \equiv a \int_{\eta_{in}}^{\eta} c_\nu(\eta) d\eta$$

Comoving horizon scale

Free streaming horizon is the key physical quantity however this role is also taken in the literature by $2\pi/k_{nr}$

NOTE: Neutrinos in the keV range will become non-relativistic in the RD era where $c_\nu \sim 1/a \sim 1/\eta$ and $H \sim \eta^{-2}$ Thus free streaming scale increases like η. While after equality it will decrease. Maximum is in this case reached between equality and non-relativistic transition In this case d_{FS} can be much larger than $1/k_{nr}$ (and grows logarithmically)

Vlasov (neutrino) equation

$$\frac{Df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = 0$$

1. Perturb f and keep linear order only
2. Fill with the metric
3. Go to Fourier

$$\frac{\partial f}{\partial \tau} + \frac{d\mathbf{x}}{d\tau}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{d\mathbf{q}}{d\tau}\cdot\frac{\partial f}{\partial \mathbf{q}} = C[f]$$

General case with non-zero Non-gravitational interactions C[f]

$$\dot{\Psi} - i\,\frac{q}{\epsilon}\,(\vec{k}\cdot\hat{n})\,\Psi = -\left(\dot{\psi} + i\,\frac{\epsilon}{q}\,(\vec{k}\cdot\hat{n})\,\phi\right)\frac{\partial \ln f_0}{\partial \ln q}\,.$$

Neutrino phase space

Metric

Vlasov (neutrino) equation in MD regime

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$$\frac{dq}{d\tau} = q\dot{\psi} + (q^2 + a^2 m^2)^{1/2} \hat{n}_i \, \partial_i \phi$$ Geodesic equation

$$\Psi(\vec{k}, q, \hat{n}, \tau) = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l(\vec{k}, q, \tau) P_l(\hat{k} \cdot \hat{n}) \, .$$ Legendre polynomials P_l

Then, the Vlasov equation becomes

$$\dot{\Psi}_0 = \frac{q\,k}{\epsilon} \Psi_1 - \dot{\psi} \frac{\partial \ln f_0}{\partial \ln q} \, ,$$
$$\dot{\Psi}_1 = \frac{q\,k}{3\,\epsilon} (\Psi_2 - 2\Psi_0) + \frac{\epsilon\,k}{3\,q} \phi \frac{\partial \ln f_0}{\partial \ln q} \, ,$$
$$\dot{\Psi}_l = \frac{q\,k}{(2l+1)\,\epsilon} \left[(l+1)\Psi_{l+1} - l\Psi_{l-1} \right] \, , \quad l \geq 2 \, .$$

Infinite series of differential equations
With multipoles related to physical quantities

$$\delta\rho_\nu = 4\pi a^{-4} \int q^2 dq \, \epsilon f_0(q) \Psi_0 \, ,$$
$$\delta p_\nu = \frac{4\pi}{3} a^{-4} \int q^2 dq \, \frac{q^2}{\epsilon} f_0(q) \Psi_0 \, ,$$

$$(\bar{\rho}_\nu + \bar{p}_\nu)\theta_\nu = 4\pi k a^{-4} \int q^2 dq \, q f_0(q) \Psi_1 \, ,$$
$$(\bar{\rho}_\nu + \bar{p}_\nu)\sigma_\nu = -\frac{8\pi}{3} a^{-4} \int q^2 dq \, \frac{q^2}{\epsilon} f_0(q) \Psi_2 \, .$$

Vlasov (neutrino) equation in MD regime - II

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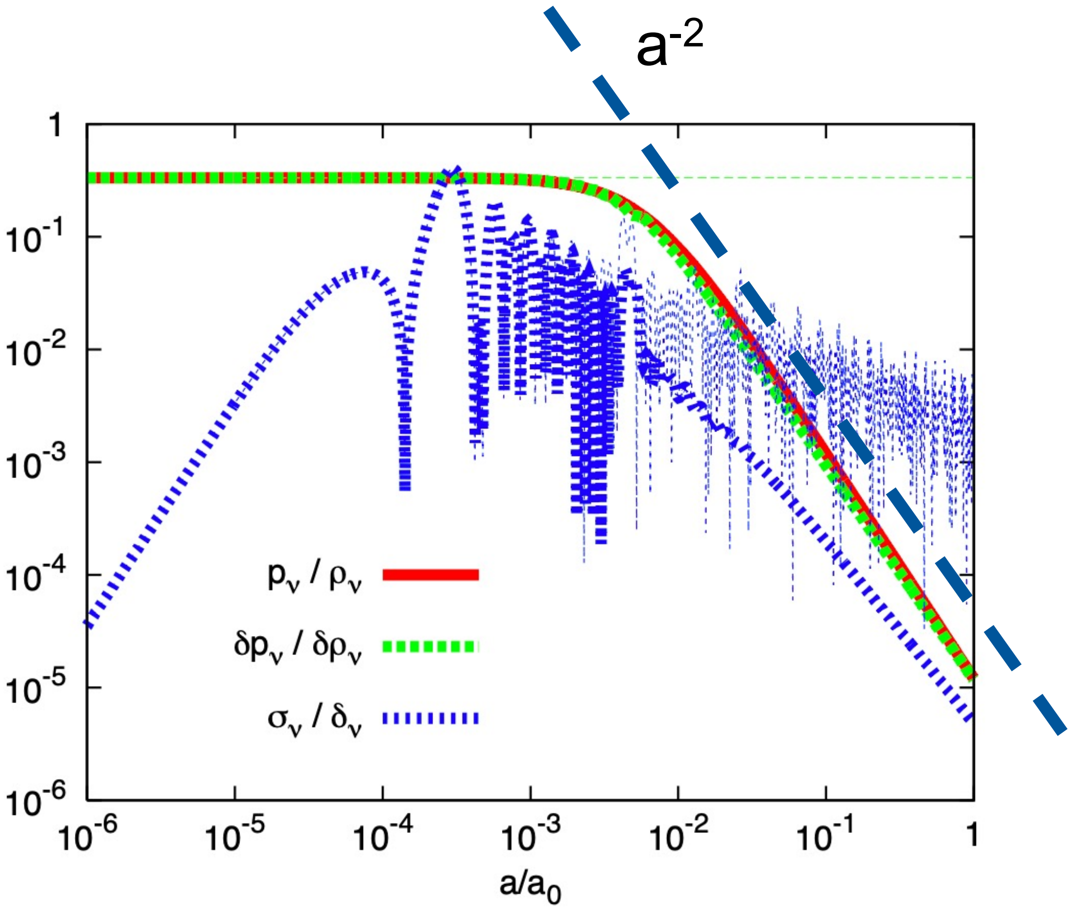


This is for the mode k=0.1 h/Mpc and m=0.1 eV using adiabatic initial conditions set by inflation as initial conditions
Thin lines are the massless case

Isotropic stress perturbation
And shear stress start to be subdominant
After NR transition

Damping is clearly visible
After non-relativistic transition
$a/a_0=5 \times 10^{-3}$

CMBFAST, CLASS, CAMB
Public available codes

Vlasov (neutrino) equation in MD regime - III

Well in the matter dominated regime, things get simpler

$$\dot{\delta}_\nu = \theta_\nu + 3\dot{\psi} \ ,$$
$$\dot{\theta}_\nu = -\frac{\dot{a}}{a}\theta_\nu - k^2\phi \ .$$

ψ and ϕ ~ const and $a \sim \tau^2$

$$\ddot{\delta}_\nu + \frac{\dot{a}}{a}\dot{\delta}_\nu = -k^2\phi + 3(\ddot{\psi} + \frac{\dot{a}}{a}\dot{\psi})$$

$$\delta_\nu = A\ \ln\tau + B - \frac{(k\tau)^2}{6}\phi$$
$$= \tilde{A}\ \ln a + \tilde{B} - \frac{2}{3}\left(\frac{k}{aH}\right)^2\phi \ ,$$

Neutrinos grow like matter!

$$\delta_\nu \longrightarrow -\frac{2}{3}\left(\frac{k}{aH}\right)^2\phi \ \propto a$$

This above is solution of Poisson
Equation in a MD Universe
For $k>k_{nr}$
Note that δ_ν can grow faster than ~ a
For a short time due to the ln a term

Vlasov (neutrino) equation in MD regime - IV

From the above equations and neglecting neutrinos' backreaction on cdm

$$P(k) = \left\langle \left(\frac{\delta\rho_{\rm cdm} + \delta\rho_{\rm b} + \delta\rho_\nu}{\rho_{\rm cdm} + \rho_{\rm b} + \rho_\nu} \right)^2 \right\rangle$$

$$= \left\langle \left(\frac{\Omega_{\rm cdm}\,\delta_{\rm cdm} + \Omega_{\rm b}\,\delta_{\rm b} + \Omega_\nu\,\delta_\nu}{\Omega_{\rm cdm} + \Omega_{\rm b} + \Omega_\nu} \right)^2 \right\rangle$$

$$= \begin{cases} \langle \delta^2_{\rm cdm} \rangle & \text{for} \quad k < k_{\rm nr}, \\ [1 - \Omega_\nu/\Omega_{\rm m}]^2 \langle \delta^2_{\rm cdm} \rangle & \text{for} \quad k \gg k_{\rm nr}, \end{cases}$$

Factor in front of Pcdm is ~ $2f_{\nu}$ ...however…. We are lucky the actual effect will be 8 times larger… why?

Neutrinos backreactions

1. Effects from homogenous pressure and density → Hubble expansion
2. Gravitational back-reaction on metric perturbations through modification of energy-momentum tensor

To check for 1. we can omit the $\delta\nu$ in Poisson equation

$$\ddot{\delta}_{\rm cdm} + \frac{\dot{a}}{a}\,\dot{\delta}_{\rm cdm} = -k^2\phi + 3(\ddot{\psi} + \frac{\dot{a}}{a}\dot{\psi})$$

$$\ddot{\delta}_{\rm cdm} + \frac{2}{\tau}\,\dot{\delta}_{\rm cdm} - \frac{6}{\tau^2}\,\delta_{\rm cdm} = 0\ ,$$

$$f_\nu \equiv \frac{\rho_\nu}{(\rho_{\rm cdm} + \rho_{\rm b} + \rho_\nu)} = \frac{\Omega_\nu}{\Omega_{\rm m}}$$

$$\ddot{\delta}_{\rm cdm} + \frac{2}{\tau}\,\dot{\delta}_{\rm cdm} - \frac{6}{\tau^2}(1 - f_\nu)\,\delta_{\rm cdm} = 0$$

$$\delta_{\rm cdm} \propto \tau^{2p}$$

$$\delta_{\rm cdm} \propto a^{p_+} \simeq a^{1-\frac{3}{5}f_\nu}$$

$$-k^2\psi \propto a^{p_+-1} \simeq a^{-\frac{3}{5}f_\nu}$$ Poisson eq.

$$\delta_{\rm cdm} \propto [a\,g(a)]^{p_+} \simeq [a\,g(a)]^{1-\frac{3}{5}f_\nu}\ .$$

Matter power spectrum from massive vs massless vs

1. Large scales k < knr

2. k>> knr and k >> keq for a< anr $\delta^{f_\nu}_{\rm cdm}[a] = \delta^{f_\nu=0}_{\rm cdm}[(1-f_\nu)\,a]$ $\quad a^{f_\nu}_{\rm eq}/a^{f_\nu=0}_{\rm eq} = (1-f_\nu)^{-1}$

$$\delta^{f_\nu}_{\rm cdm}[a_0] = \left(\frac{a_0\, g(a_0)}{a_{\rm nr}}\right)^{1-\frac{3}{5}f_\nu} \delta^{f_\nu}_{\rm cdm}[a_{\rm nr}]$$

$$\frac{\delta^{f_\nu}_{\rm cdm}[a_0]}{\delta^{f_\nu=0}_{\rm cdm}[a_0]} = (1-f_\nu)^{1/2}\left(\frac{a_0\, g(a_0)}{a_{\rm nr}}\right)^{-\frac{3}{5}f_\nu}$$

$$\frac{P(k)^{f_\nu}}{P(k)^{f_\nu=0}} = (1-f_\nu)^3\left(\frac{a_0\, g(a_0)}{a_{\rm nr}}\right)^{-\frac{6}{5}f_\nu} = (1-f_\nu)^3\left[1.9\times 10^5\, g(a_0)\,\omega_{\rm m}\, f_\nu/N_\nu\right]^{-\frac{6}{5}f_\nu}$$

$$\frac{P(k)^{f_\nu}}{P(k)^{f_\nu=0}} \simeq -8\, f_\nu\ .$$

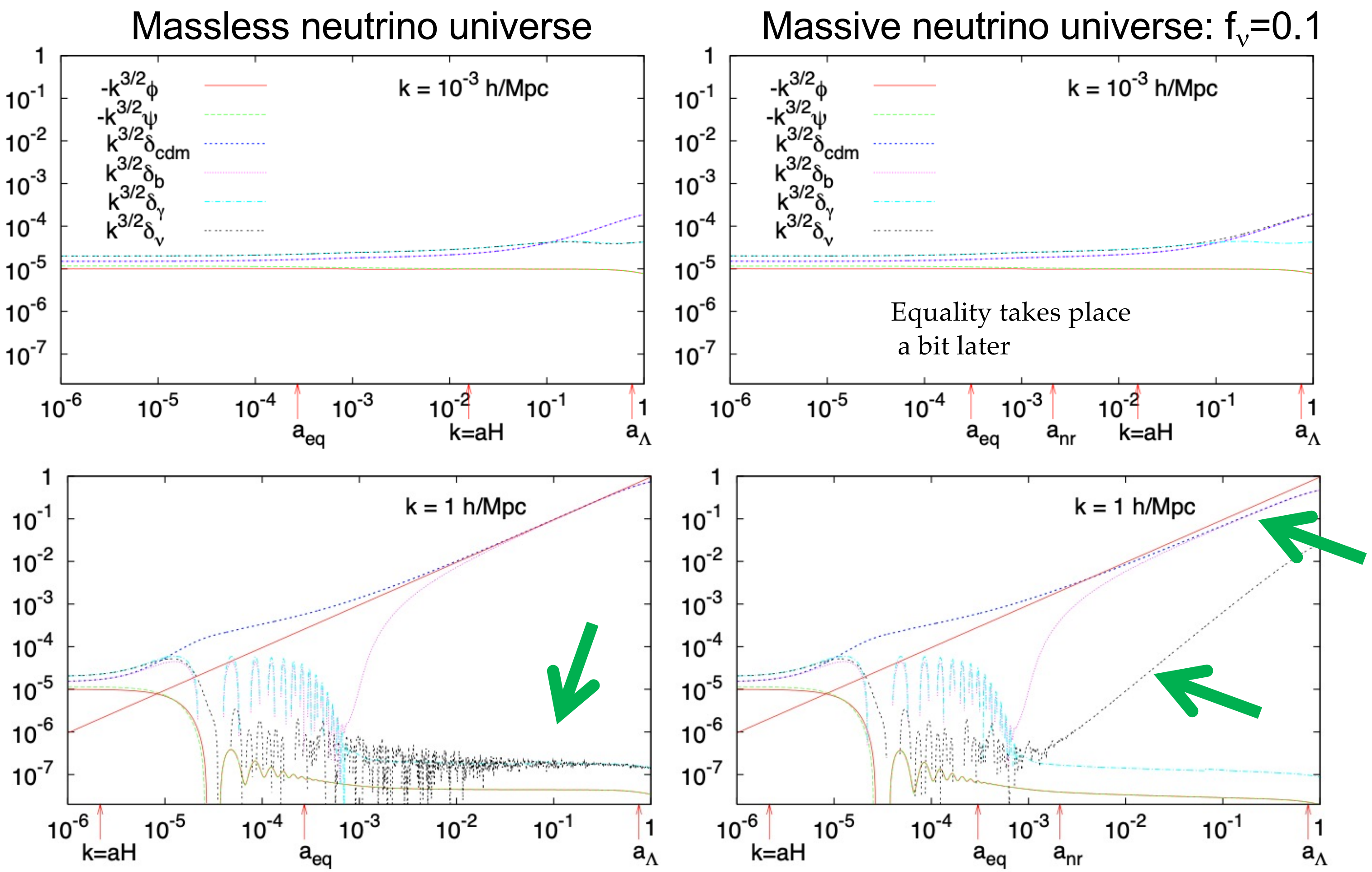
Small few % effect? Yes!
But "integrated" through structure formation era i.e. P(k) is P(k,z)

But… is it really small?---> ask also **Enzo Branchini!**

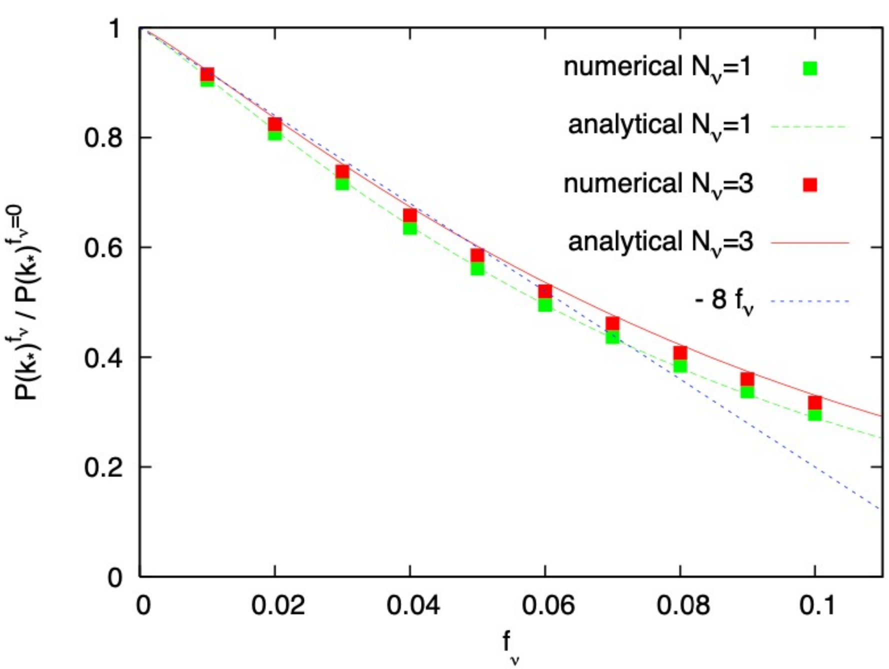
Matter power spectrum from massive vs massless vs

Matteo Viel

In practice: numerical solutions → popular Boltzmann solvers like CMBFAST and CLASS



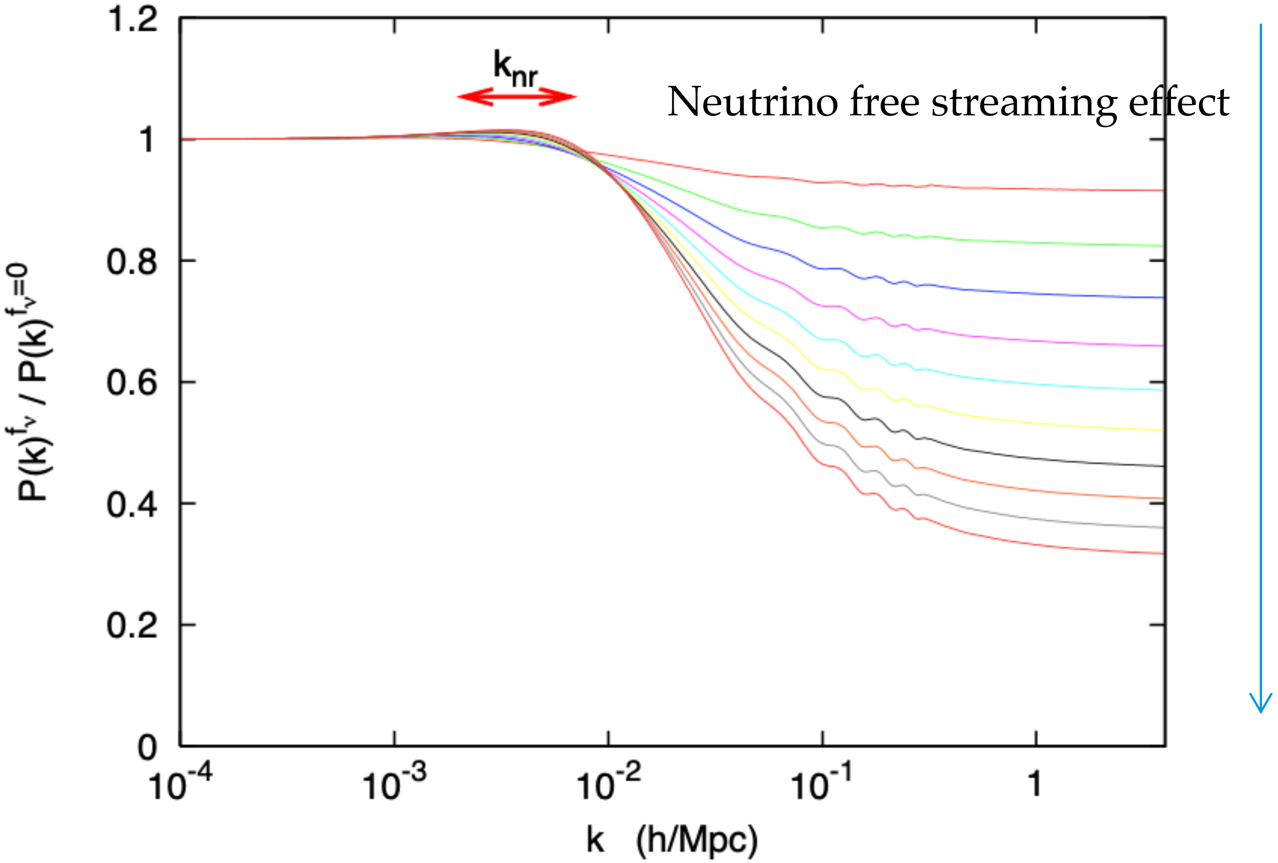
Matter power spectrum from massive vs massless vs



Suppression factor computed today at k=10 h/Mpc

Little sensitivity to the mass splitting

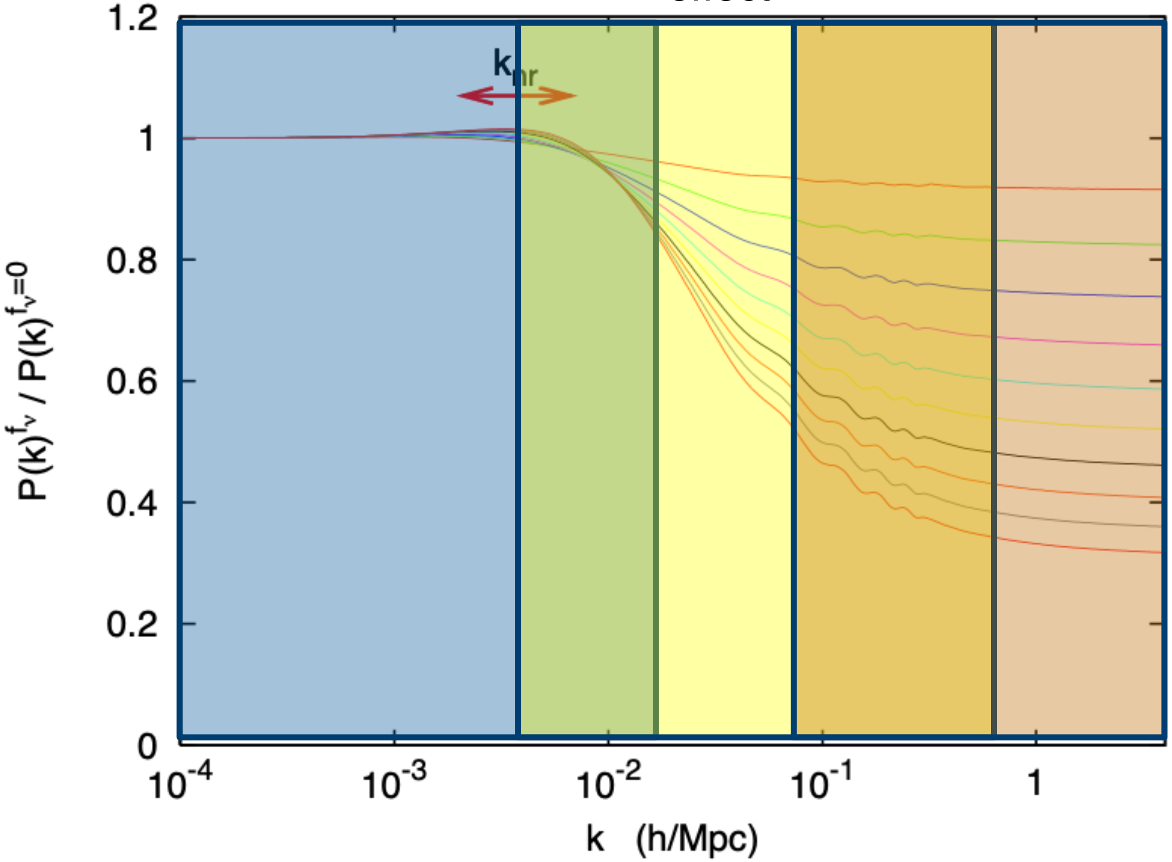
Matter power spectrum from massive vs massless vs



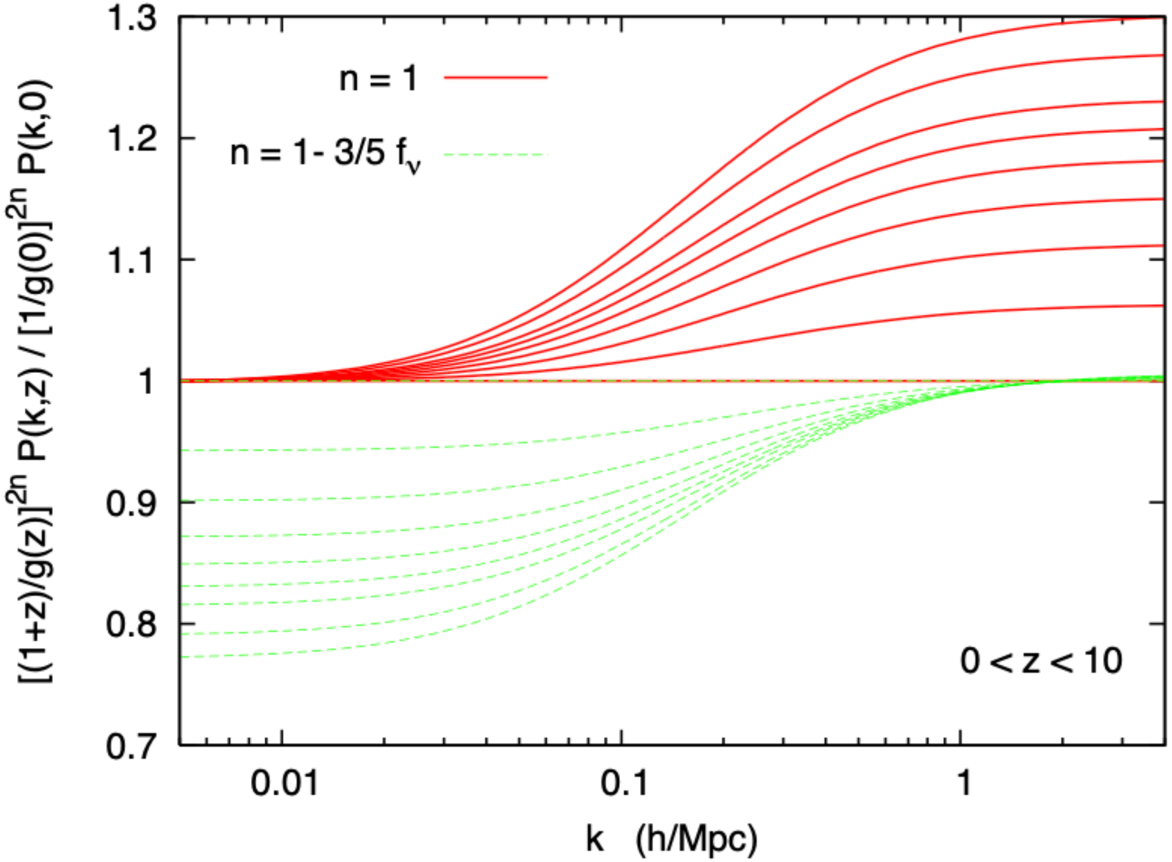
From 0.15 eV to 1.5eV total neutrino mass

Matter power spectrum from massive vs massless vs

Neutirino free streaming effect

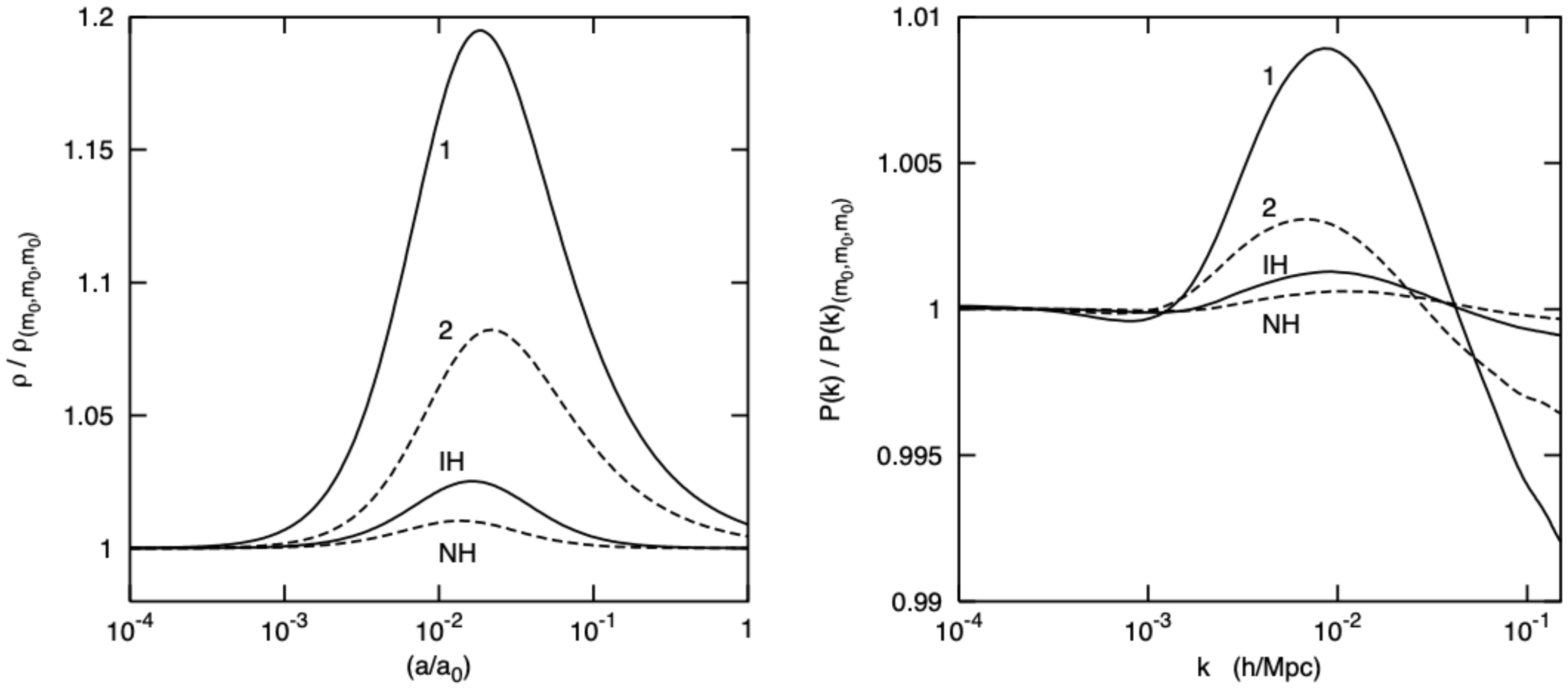


Matter power spectrum from massive vs massless vs



Matter power spectrum from massive vs massless vs

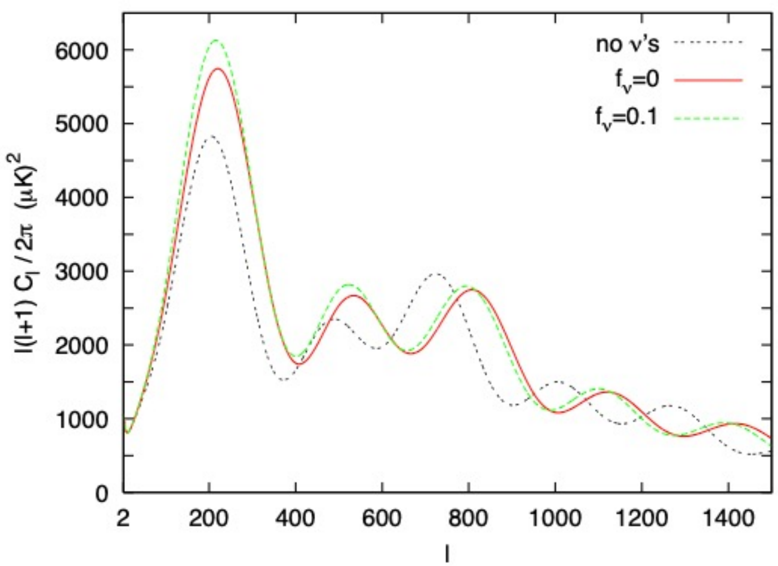
MASS SPLITTING

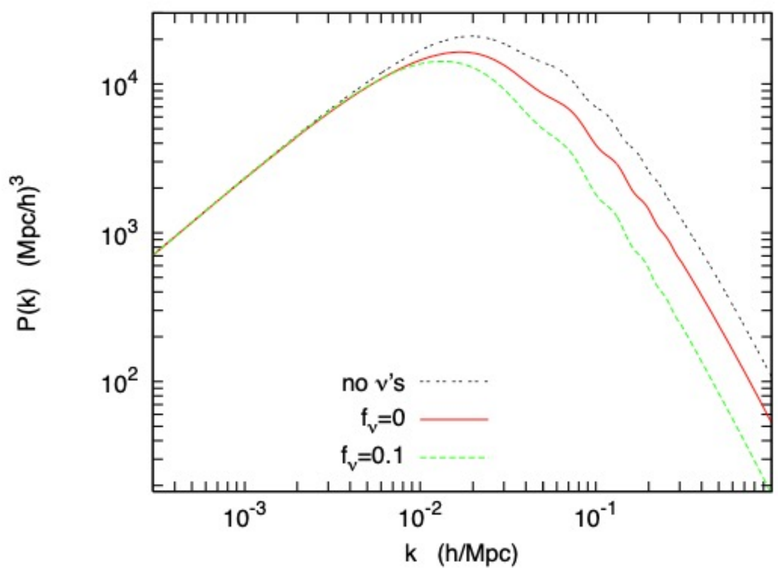


TOTAL MASS = 0.12 eV

Matter power spectrum from massive vs massless vs

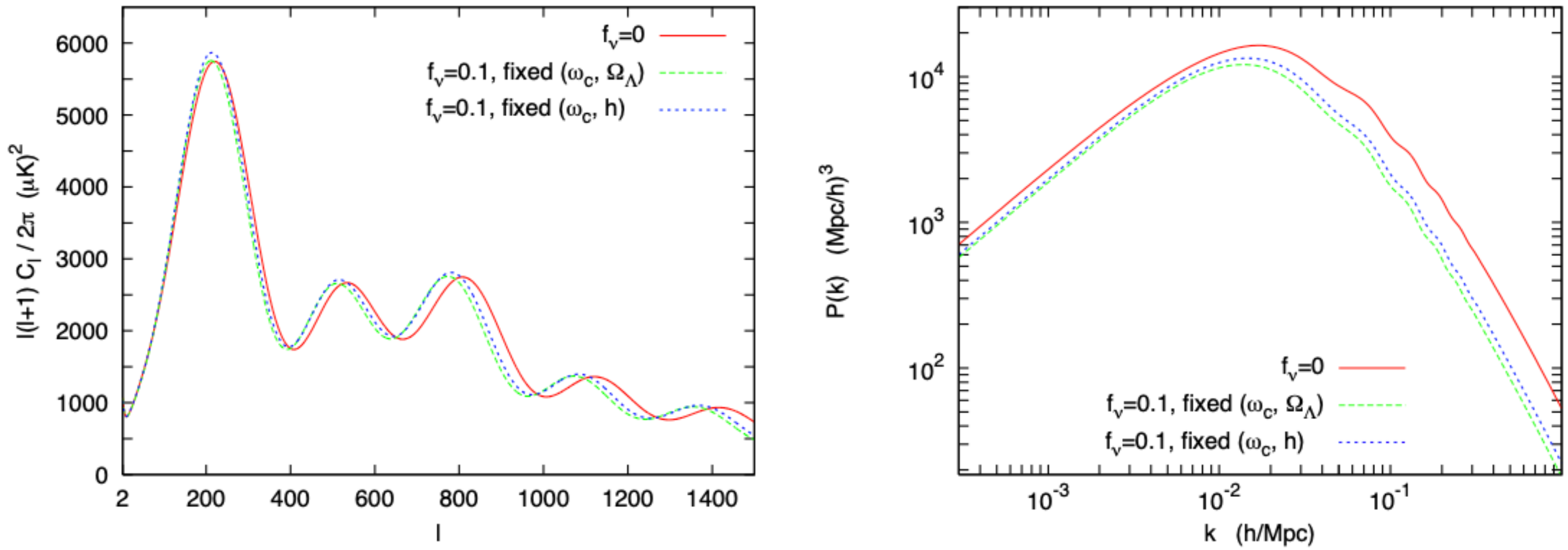
Matteo Viel





For CMB: Douglas Scott
Paolo Natoli

Matter power spectrum from massive vs massless vs



For CMB: Douglas Scott
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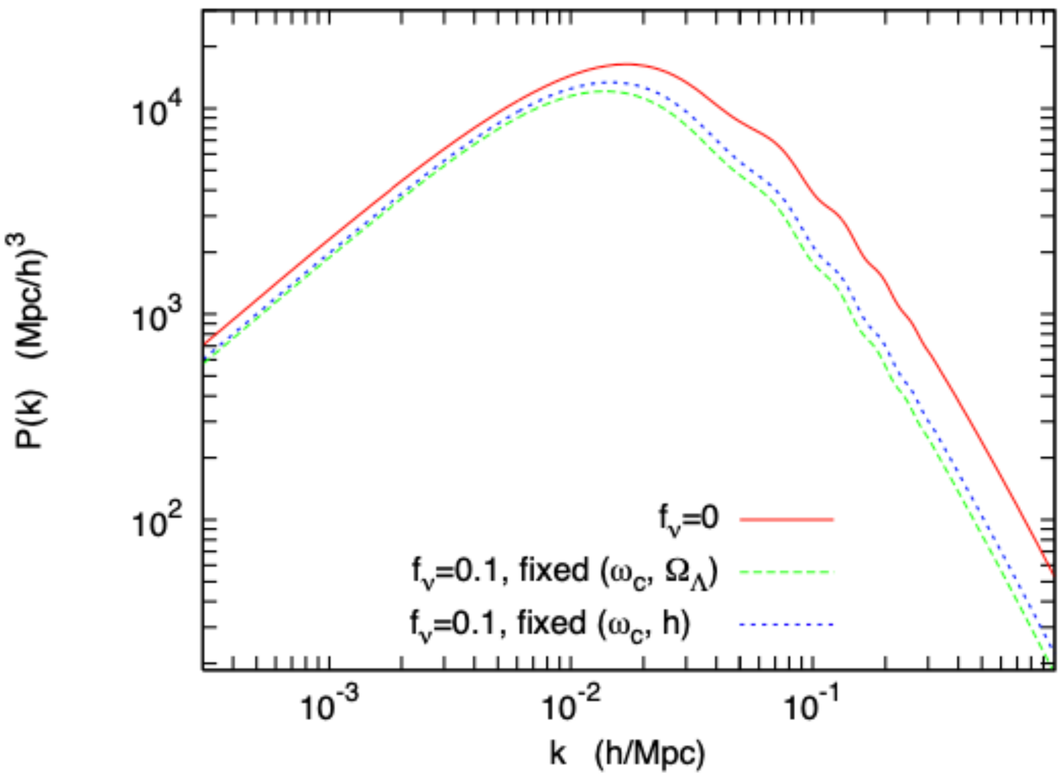
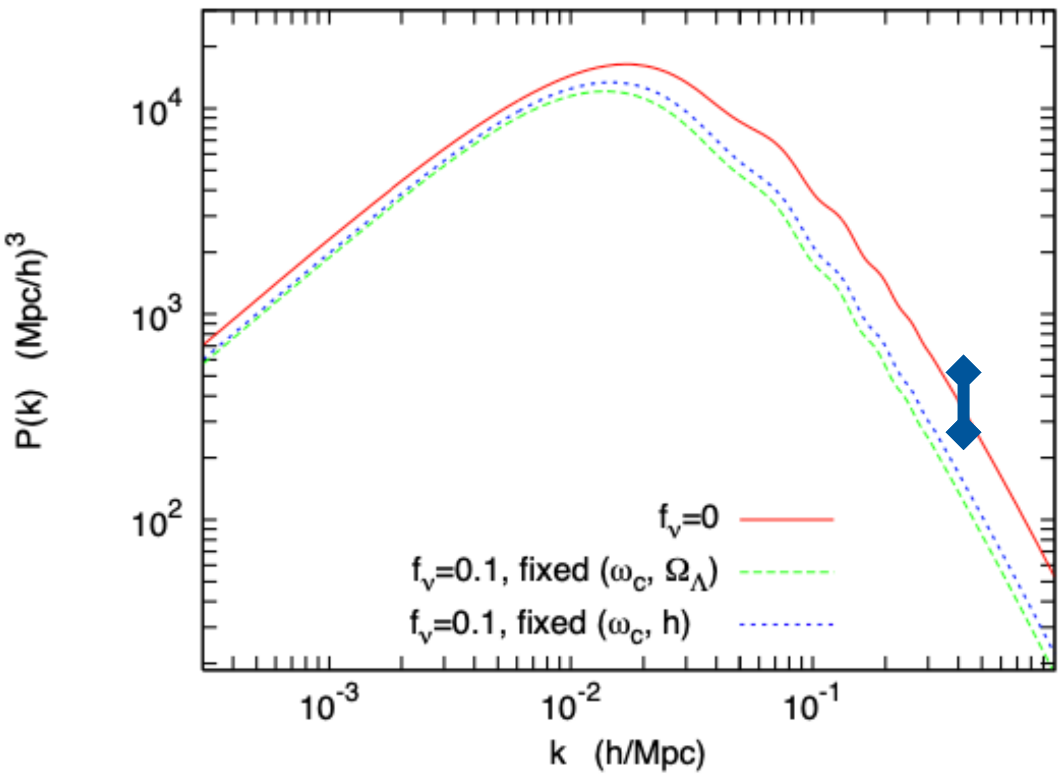
Matter power spectrum from massive vs massless vs

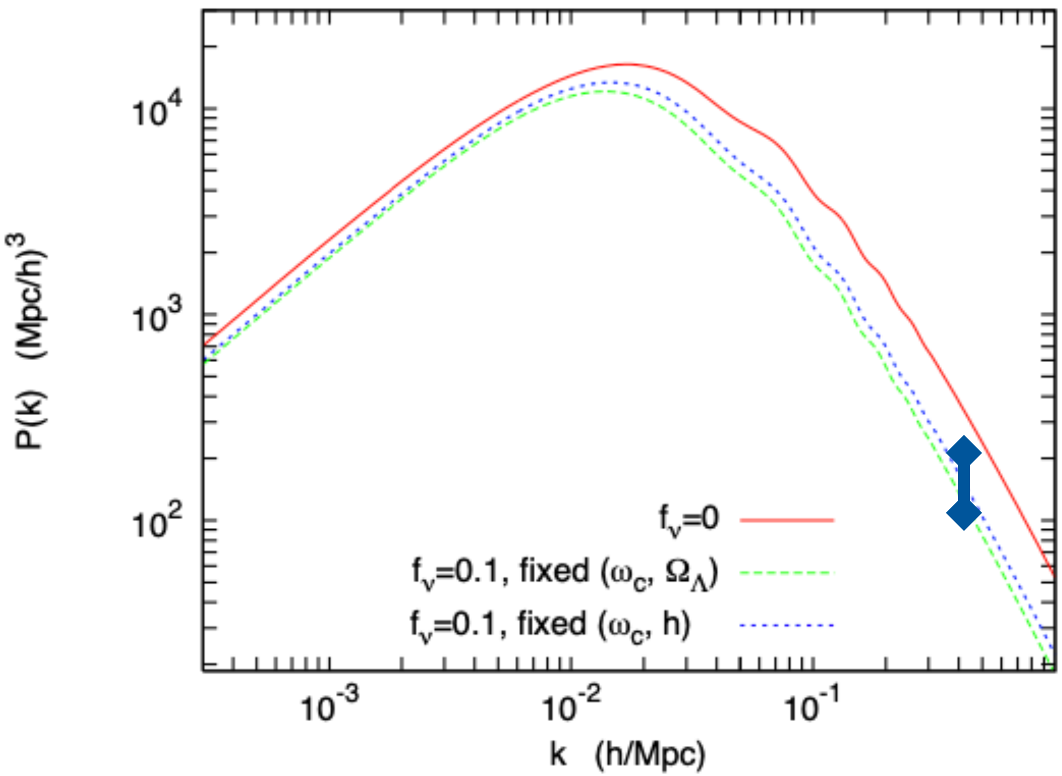


Matter power spectrum from massive vs massless vs



Matter power spectrum from massive vs massless vs

Matteo Viel



Recap - key moments

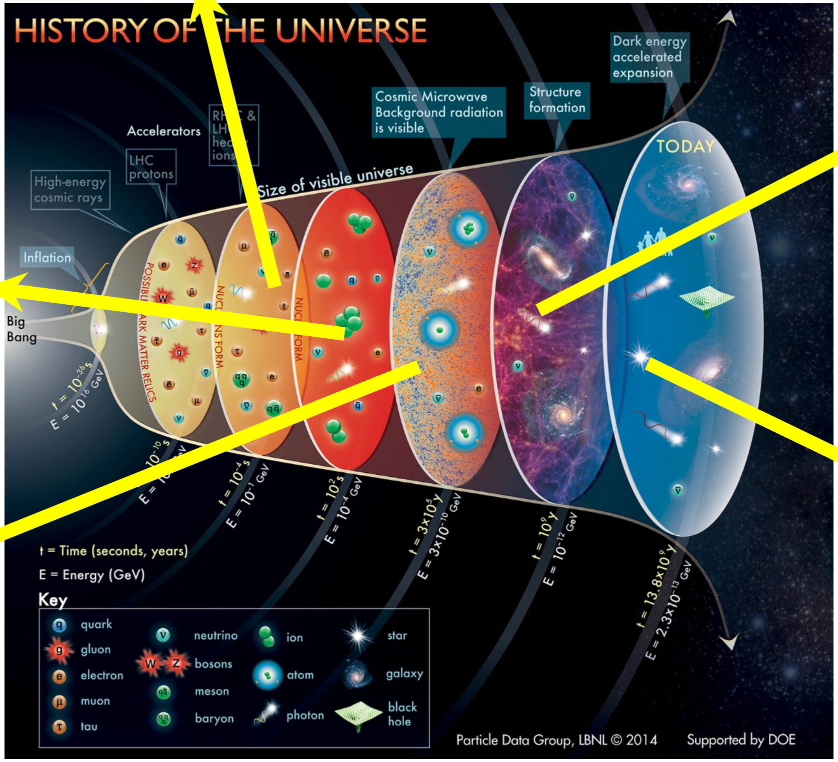
1. Relativistic neutrino contribution to cosmic expansion

2. Neutrino free streaming slows down CMB photon clustering

3. Metric fluctuations During non-relativistic neutrino transition (early ISW)

4. Neutrino free streaming slows down ordinary matter clustering

5. Non-relativistic neutrino contribution to late expansion rate



BBN CMB LSS

Recap - key take home messages

Matteo Viel

1. Neutrino number density is large, like photons, in terms of number density is the second most abundant species
   $n_{\gamma} \sim n_{\nu} \sim 10^{10}\, n_{\text{atoms, e-}}$

2. Unlike other particles they become non-relativistic **after** decoupling and do not annihilate

3. Looking at the whole Universe from large to small scales they can be probed