Cosmological Neutrinos

Matteo Viel - SISSA (Trieste, Italy)

Varenna – PhD School – 02/07/23

Plan

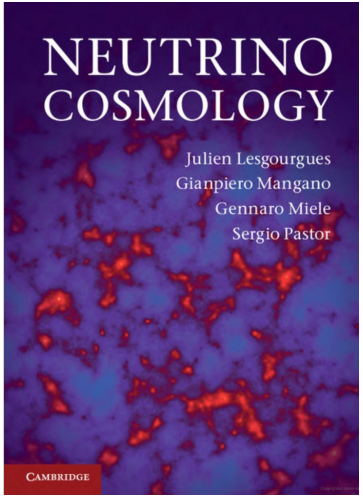
- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Some references

Lesgourgues and Pastor 2006 review on the arXiv

Lesgourgues, Mangano, Miele, Pastor “Neutrino Cosmology” 2013 Cambridge University Press

Ma & Bertschinger https://arxiv.org/pdf/astro-ph/9506072.pdf



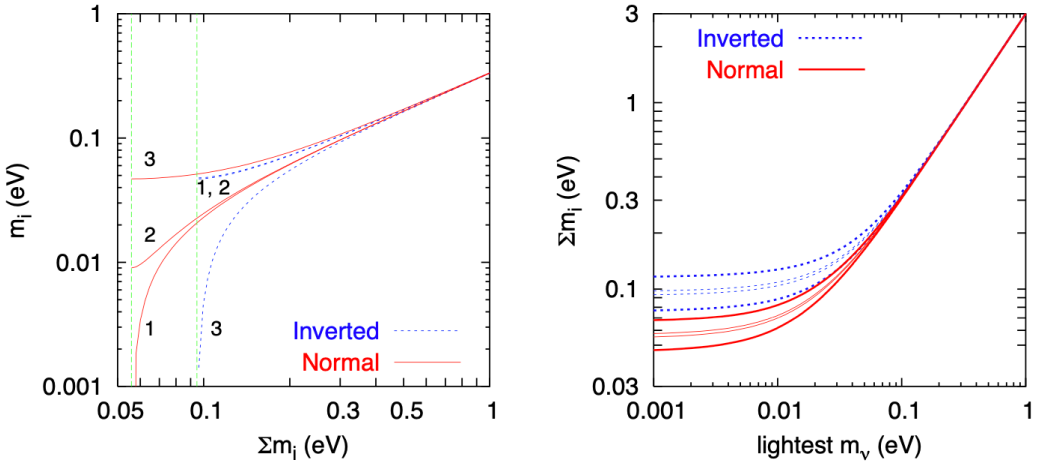
Linear Perturbation theory

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Facts

Boundary conditions from **particle physics**

$$0.056\,(0.095)\ \mathrm{eV} \lesssim \sum_i m_i \lesssim 6\ \mathrm{eV}$$



We will see that cosmology will be sensitive to total neutrino mass

The neutrino background: neutrino decoupling

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Alpher, Follin, Herman 1953
CνB

Firm established prediction of the standard Big Bang model

$$f_{\rm eq}(p) = \left[\exp\left(\frac{p-\mu_\nu}{T}\right)+1\right]^{-1}$$ With chemical potential ~ 0

Weak interaction rate

$$\Gamma_\nu = \langle \sigma_\nu \, n_\nu \rangle$$

Hubble parameter

$$H = \sqrt{\frac{8\pi\rho}{3M_P^2}}$$

T_{dec} ~ MeV [at 1 sec]

After decoupling: f_{eq} is preserved because T and p scale as $1/a$
No dependence on the mass

This means that momentum distribution is exact even in the epochs of structure formation!

The neutrino background: energy densities

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After neutrino decoupling photon temperature drops below electron mass, e+e- annihilation heat the plasma [we are at T~0.5 MeV or so]

$$n_\nu = \frac{3}{11}\, n_\gamma = \frac{6\zeta(3)}{11\pi^2}\, T_\gamma^3$$

n_ν / n_γ = 0.68
At any time after electron
Positron annihilation

$$\rho_\nu(m_\nu \ll T_\nu) = \frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3} T_\gamma^4$$
$$\rho_\nu(m_\nu \gg T_\nu) = m_\nu n_\nu \ .$$

Two well-defined limits for matter and radiation

Note: there are small non-thermal distortions in the neutrino FD spectrum and a slight increase in the photon neutrino temperature due to relic interactions of e+ e- with neutrinos [e.g. Dolgov (2002)]

The neutrino background: today

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Extrapolating to today:

$n_\nu = 339.5\ \mathrm{cm}^{-3}$ average

Note: Clustering will change this number

$T_\gamma = 2.73\ \mathrm{K} \rightarrow T_\nu = 1.95\ \mathrm{K}$

$\langle p \rangle = 1.7 \times 10^{-4}\ \mathrm{eV} = 3.15\ T_\nu$

$$z_{\rm nr} = \left(\frac{m_\nu}{5.28\times10^{-4}{\rm eV}}\right)\left(\frac{T_\nu^a}{T_\nu}\right) - 1$$

$$\Omega_\nu h^2 = \frac{m_\nu}{93.14\,{\rm eV}}$$

$\Omega_\nu h^2 \geq 6 \times 10^{-4}$ (NO), or $\geq 10^{-3}$ (IO)

$\Omega_\nu > 0.5\%$ of matter components – sub-dominant matter component
Hot DM
5-th most abundant Universe component by energy
2nd most abundant by number density

Very small numbers → direct detection difficult

The neutrino background – radiation era

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$$\rho_{\rm R} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_\gamma$$

Extra radiation contribution

Neff= energy density of neutrinos + other light particles / energy density of 1neutrino family neglecting e+e-> ν

Neff is accurately theoretically estimated to be 3.044 and also measured from cosmological observations

$N_{\rm eff} = 3.00^{+0.57}_{-0.53}$ (95 %, *Planck* TT+lowE),

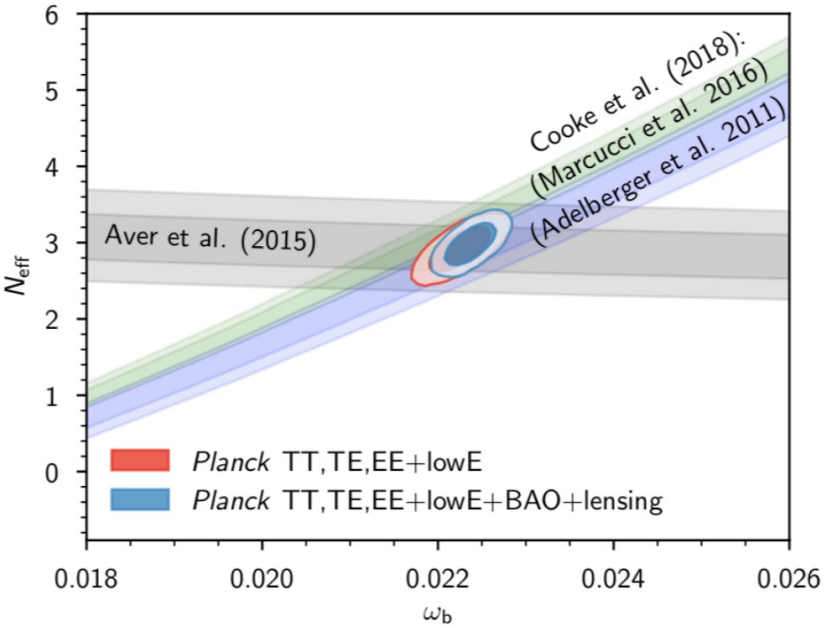
$N_{\rm eff} = 2.92^{+0.36}_{-0.37}$ (95 %, *Planck* TT,TE,EE+lowE),

From CMB Planck 2018

Also BBN provides constraints and the error bar can be reduced by a factor 2 see Yeh+22

Convincing proof of existence of Cosmic neutrino background

N_{eff}



Planck collaboration VI, 2018

Remarkable success of cosmological data in probing particle dark matter (including Non-standard heat injection in the primordial Universe)

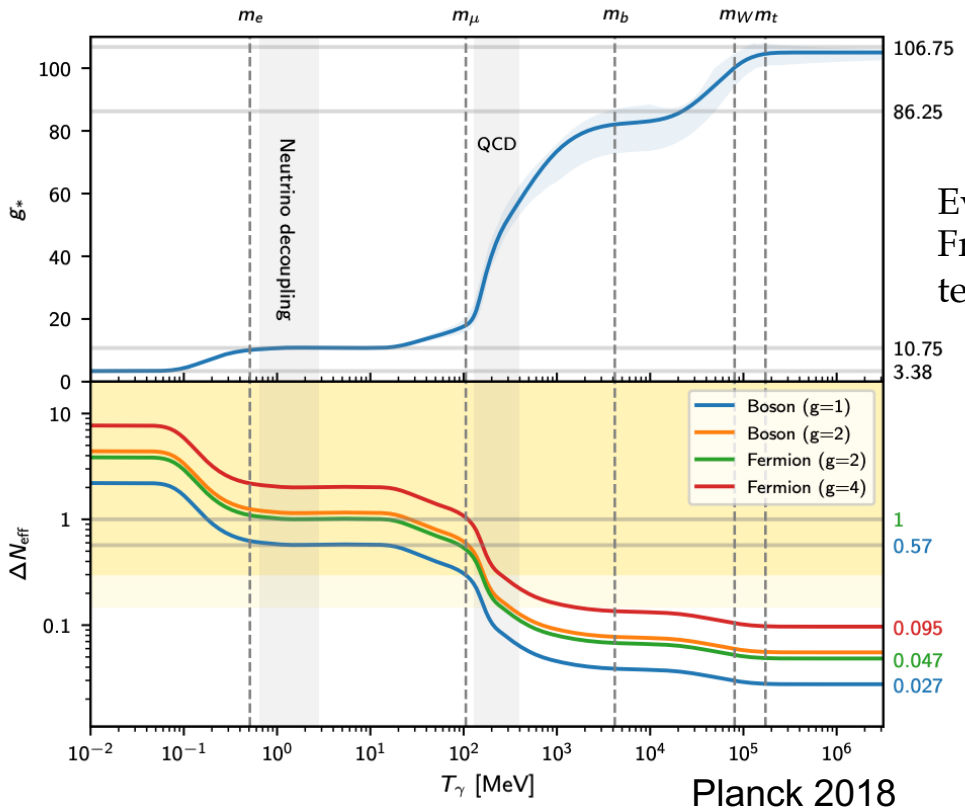
Neff now detected at >10-20σ

But there is more:
Detected anisotropies/perturbations in the fluid and their imprint on the CMB (especially polarization)

Everything consistent with a Relativistic fluid with no viscosity!

N_{eff} to test particle (new) physics

Constraints on additional relativistic particles



Planck 2018

$$\Delta N_{\rm eff} = g \left[\frac{43}{4\, g_s}\right]^{4/3} \times \begin{cases} 4/7 & \text{boson,} \\ 1/2 & \text{fermion,} \end{cases}$$

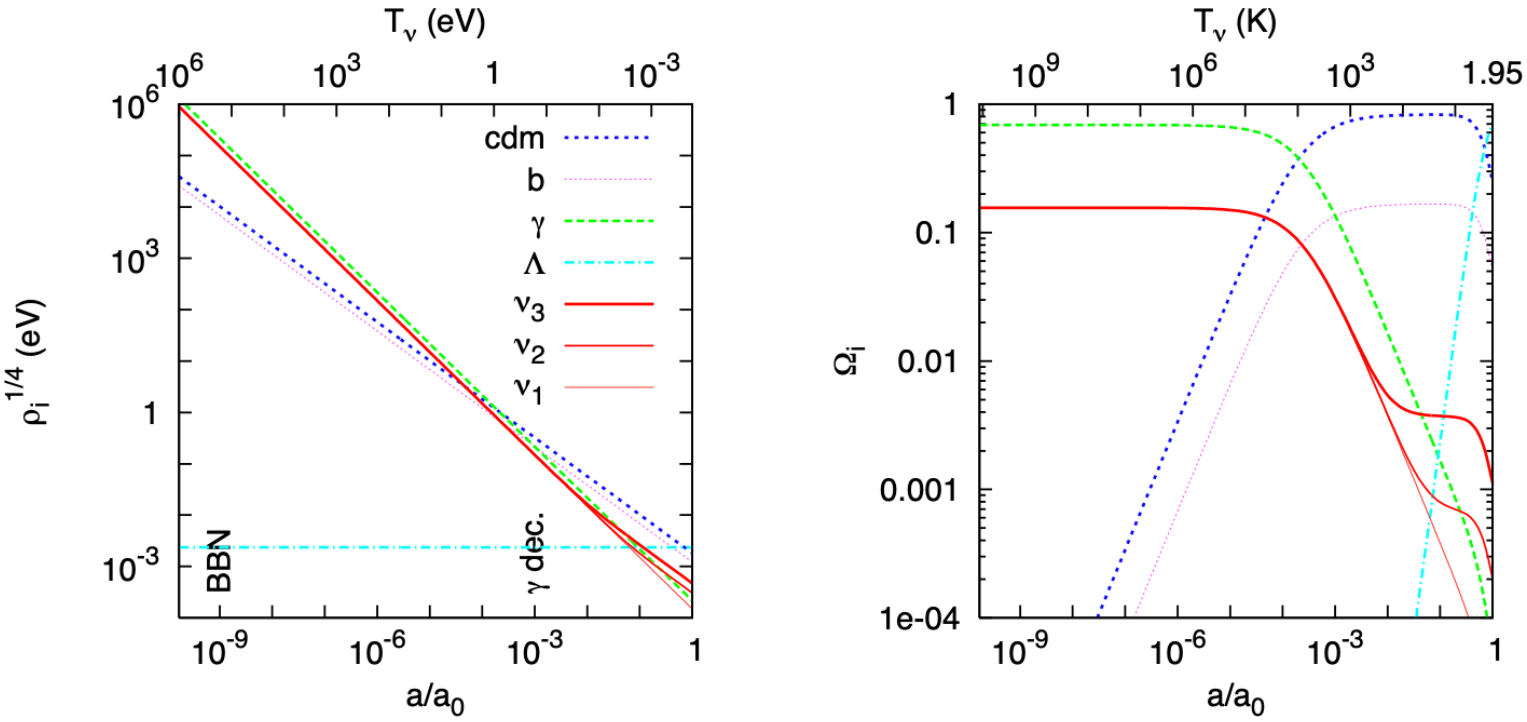
Fully thermalized relics

Evolution of effective degrees of Freedom for SM particles vs photon temperature

Expected ΔN_{eff} today

Evolution of energy densities in a neutrino Universe

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Normal hierarchy with m_1=0 eV, m_2=0.009 eV, m_3= 0.05eV

The perturbed Universe

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1. The metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(\tau)[(1+2\phi)d\tau^2 - (1-2\psi)\delta_{ij}dx^i dx^j]$$

2. The tensor

$\delta T^0_0 = \delta\rho$, Energy density perturbation

$\delta T^0_i = (\bar{\rho}+\bar{p})v_i^{||}$, Longitudinal component of velocity field

$\delta T^i_j = -\delta p\,\delta^i_j + \Sigma^{i||}_j$, dp pressure perturabtion, traceless and longitudinal component of the 3x3 tensor

3. New Variables

Velocity divergence $\theta \equiv \sum_i \partial_i v_i = \nabla^2\tilde{v}$,

Shear (anisotropic) stress $(\bar{\rho}+\bar{p})\nabla^2\sigma \equiv -\sum_{i,j}(\partial_i\partial_j - \frac{1}{3}\nabla^2\delta_{ij})\Sigma^{i||}_j = -\frac{2}{3}\nabla^4\tilde{\sigma}$

The perturbed Universe - II

4. Perturbed Einstein equation (which imply conservation of total energy momentum-tensor)

$$\delta G^0_0 = 2a^{-2}\left\{-3\left(\frac{\dot a}{a}\right)^2\phi - 3\frac{\dot a}{a}\dot\psi + \nabla^2\psi\right\} = 8\pi G\ \delta\rho\ ,$$

$$\delta G^0_i = 2a^{-2}\partial_i\left\{\frac{\dot a}{a}\phi + \dot\psi\right\} = 8\pi G\ (\bar\rho + \bar p)\ v_i\ ,$$

$$\delta G^i_j = -2a^{-2}\left\{\left[\left(2\frac{\ddot a}{a} - \left(\frac{\dot a}{a}\right)^2\right)\phi + \frac{\dot a}{a}(\dot\phi + 2\dot\psi) + \ddot\psi + \frac{1}{3}\nabla^2(\phi-\psi)\right]\delta^i_j\right.$$
$$\left. -\frac{1}{2}\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i_j\right)(\phi-\psi)\right\} = 8\pi G\ (-\delta p\ \delta^i_j + \Sigma^i_j)$$

5. Perturbed Einstein equations + change of variable + let's move to Fourier space

$$-3\left(\frac{\dot a}{a}\right)^2\phi - 3\frac{\dot a}{a}\dot\psi - k^2\psi = 4\pi G\ a^2\ \bar\rho\ \delta\ ,$$

$$-k^2\left(\frac{\dot a}{a}\phi + \dot\psi\right) = 4\pi G\ a^2\ (\bar\rho + \bar p)\ \theta\ ,$$

$$\left(2\frac{\ddot a}{a} - \left(\frac{\dot a}{a}\right)^2\right)\phi + \frac{\dot a}{a}(\dot\phi + 2\dot\psi) + \ddot\psi - \frac{k^2}{3}(\phi-\psi) = 4\pi G\ a^2\ \delta p\ ,$$

$$k^2(\phi-\psi) = 12\pi G\ a^2\ (\bar\rho + \bar p)\ \sigma$$

The perturbed Universe - III

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6. ….but we are dealing with fluids: continuity for each fluid component

$$\dot{\delta} = (1+w)(\theta + 3\dot{\psi})$$

7. … and Euler for **each** fluid component

$$\dot{\theta} = \frac{\dot{a}}{a}(3w-1)\theta - \frac{\dot{w}}{1+w}\theta - k^2\phi - k^2\sigma - \frac{w}{1+w}k^2\delta$$

$w = \bar{p}/\bar{\rho} = \delta p/\delta\rho.$

8. … and a perfect fluid → energy-momentum tensor is diagonal and isotropic

$$T^{\mu\nu} = -p\ g^{\mu\nu} + (\rho + p)U^\mu U^\nu$$

$U^\mu = dx^\mu/[a(1+\phi)d\tau]$ and obtain:

$$U^\mu = \left(a^{-1}[1-\phi],\, a^{-1}v^i\right)\ , \qquad T^0_0 = \rho\ , \quad T^i_0 = v^i\ , \quad T^i_i = -p\ ,$$

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9. Now solve Einstein equations in a **neutrinoless** Univers with the perturbed energy-momentum tensor

$$\delta T^0_0 = \delta\rho_{\rm r} + \delta\rho_{\rm m} \ ,$$
$$\partial^i(\delta T^0_i) = (\bar\rho_{\rm r} + \bar p_{\rm r})\theta_{\rm r} + \bar\rho_{\rm m}\theta_{\rm m} = \frac{4}{3}\bar\rho_{\rm r}\theta_{\rm r} + \bar\rho_{\rm m}\theta_{\rm m} \ ,$$
$$\delta T^i_i = -\delta p_{\rm r} = -\frac{1}{3}\delta\rho_{\rm r}$$

At this point it is very important to define a **Jeans length**

Causal Horizon/Particle Horizon
(maximum physical
scale at which a signal can propagate)

$$d(t_i,t) = a(t)\int_{t_i}^{t} dx = a(t)\int_{t_i}^{t}\frac{v\,dt'}{a(t')}$$

Hubble radius/Particle Horizon
(for a ~ t^n and n<1)

$$R_H(t) = \frac{t}{n} \ , \qquad d_H(t \gg t_i) \simeq \frac{t}{1-n} \ .$$

Acoustic perturbations c_s
Sound Horizon c_s/H(t)

$$k_J(t) = \left(\frac{4\pi G\bar\rho(t)a^2(t)}{c_s^2(t)}\right)^{1/2} \ , \qquad \lambda_J(t) = 2\pi\frac{a(t)}{k_J(t)} = 2\pi\sqrt{\frac{2}{3}}\frac{c_s(t)}{H(t)}$$

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10. Evolution of perturbation in a fluid with perturbations propogating with sound speed cs

Two regimes

Large scales $k<k_J$ pressure unimportant

$$\ddot{\delta} + \frac{\dot{a}}{a}\dot{\delta} + (k^2 - k_J^2)\, c_s^2\, \delta = 0$$

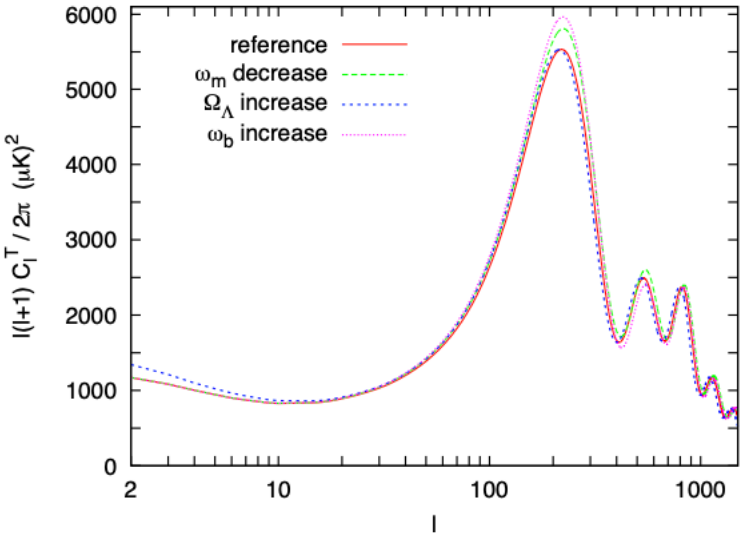
Small scales Modes with $k>k_J$ will oscillate
With frequency k x c_s (competition
Between pressure and gravity)

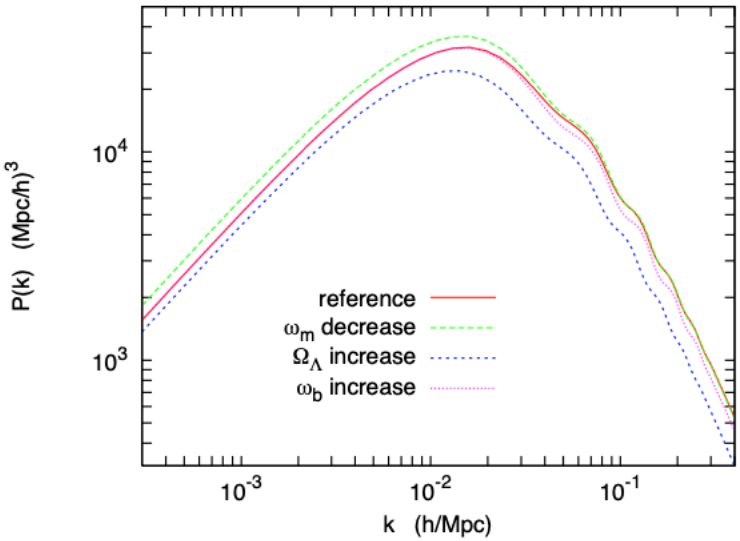
11. Jeans instability is a key ingredient for structure formation
before recombination c_s~c the photon-baryon fluid oscillates on scales smaller than lambdaJ
after recombination $c_s \rightarrow 0$ $lambda_J \rightarrow 0$ and structure can grow

The perturbed Universe - VI

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In a neutrinoless Universe





The perturbed Universe: adding neutrinos

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At least 2 neutrinos should be matter – an extra matter component, this implies that they turned non-relativistic during matter domination z<3000

$$q_i \equiv a p_i, \qquad P_i = a(1-\psi)p_i \qquad f(x^i, P_j, \tau) = f_0(P, \tau)$$

In absence of perturbation pi will decade as $1/a$ while Pi will stay constant

$$\epsilon = a(p^2 + m^2)^{1/2} = (q^2 + a^2 m^2)^{1/2}$$

$$T_0^0 = \bar{\rho}_\nu = \frac{4\pi}{a^4} \int q^2 dq\, \epsilon\, f_0(q)\ , \qquad f_0(q) = \frac{1}{e^{q/aT_\nu} + 1}$$

$$T_i^i = -\bar{p}_\nu = -\frac{4\pi}{3a^4} \int q^2 dq\, \frac{q^2}{\epsilon}\, f_0(q)$$

Spatial perturbations in the metric will induce variations in neutrino phase-space density
Depending on time, space and momentum → this will impact on the energy momentum tensor

Scalar sector of the tensor will contain now the anisotropic stress (different w.r.t. perfect fluid)

The perturbed Universe: the energy momentum tensor

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$$f(x^i, P_j, \tau) = f_0(q)[1 + \Psi(x^i, q_j, \tau)] \qquad \text{with} \qquad P_j = (1 - \Psi)q_j$$

$$T_{\mu\nu} = \int dP_1 dP_2 dP_3 \, (-g)^{-1/2} \, \frac{P_\mu P_\nu}{P^0} f(x^i, P_j, \tau)$$

$$T^0_0(x^i) = a^{-4} \int q^2 dq \, d\Omega \, \epsilon \, f_0(q) \, [1 + \Psi(x^i, q\hat{n}_j, \tau)] \, ,$$

$$T^0_i(x^i) = a^{-4} \int q^2 dq \, d\Omega \, q \, \hat{n}_i \, f_0(q) \, \Psi(x^i, q\hat{n}_j, \tau) \, ,$$

$$T^i_j(x^i) = -a^{-4} \int q^2 dq d\Omega \, \frac{q^2}{\epsilon} \, \hat{n}_i \hat{n}_j \, f_0(q) \, [1 + \Psi(x^i, q\hat{n}_j, \tau)] \, ,$$

$$\delta\rho_\nu = a^{-4} \int q^2 dq \, d\Omega \, \epsilon f_0 \Psi \, , \qquad \delta P_\nu = \frac{1}{3} a^{-4} \int q^2 dq \, d\Omega \, \frac{q^2}{\epsilon} f_0 \Psi \, , \qquad (92)$$

$$\delta T^0_{i\,\nu} = a^{-4} \int q^2 dq \, d\Omega \, q\hat{n}_i \, f_0 \Psi \, , \quad \Sigma^i_{j\,\nu} = -a^{-4} \int q^2 dq \, d\Omega \, \frac{q^2}{\epsilon} (\hat{n}_i \hat{n}_j - \frac{1}{3}\delta_{ij}) \, f_0 \Psi \, .$$

The perturbed Universe: neutrino free streaming

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As we defined the Jeans length we can now replace cs with vthermal

$$k_{FS}(t) = \left(\frac{4\pi G\bar{\rho}(t)a^2(t)}{v_{\rm th}^2(t)}\right)^{1/2}, \qquad \lambda_{FS}(t) = 2\pi\frac{a(t)}{k_{FS}(t)} = 2\pi\sqrt{\frac{2}{3}}\frac{v_{\rm th}(t)}{H(t)}$$

$$v_{\rm th} \equiv \frac{\langle p\rangle}{m} \simeq \frac{3T_\nu}{m} = \frac{3T_\nu^0}{m}\left(\frac{a_0}{a}\right) \simeq 150(1+z)\left(\frac{1\,{\rm eV}}{m}\right){\rm km\,s^{-1}}$$

$$\lambda_{FS}(t) = 7.7\frac{1+z}{\sqrt{\Omega_\Lambda+\Omega_m(1+z)^3}}\left(\frac{1\,{\rm eV}}{m}\right)h^{-1}{\rm Mpc}\ ,$$

$$k_{FS}(t) = 0.82\frac{\sqrt{\Omega_\Lambda+\Omega_m(1+z)^3}}{(1+z)^2}\left(\frac{m}{1\,{\rm eV}}\right)h\,{\rm Mpc}^{-1},$$

The perturbed Universe: neutrino free streaming - II

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These scales are physical (not comoving) and set by particle physics and have to be compared to cosmic expansion
Before non-relativistic transition $\lambda_{FS} \sim t$ and $a \sim t^{1/2}$
After non-relativistic transition free-streaming scale increases $\lambda_{FS} \sim 1/(aH) \sim t^{1/3}$
But comoving $\lambda_{FS}/a \sim t^{-1/3}$, because $a \sim t^{2/3}$, comoving free streaming scale decreases!

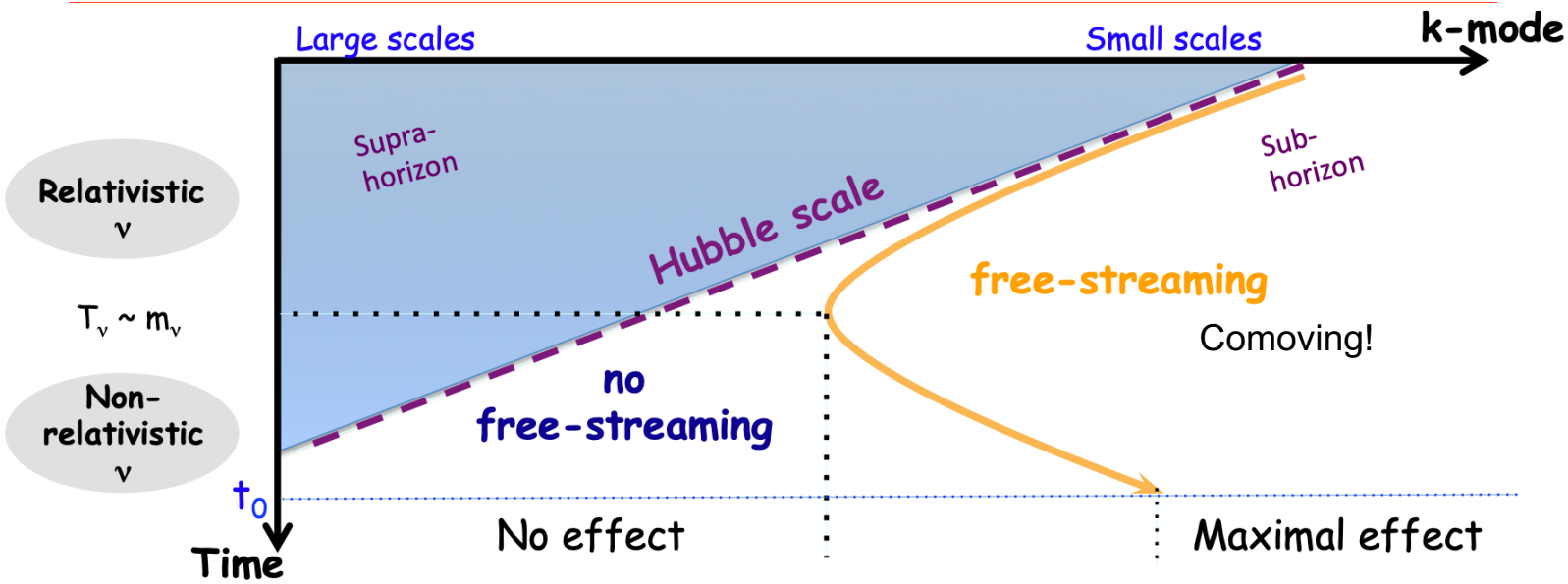
Thus, if neutrinos become relativistic during MD the comoving free-streaming scale passes through a minimum at $k=k_{nr}$ when $m=\langle p\rangle=3T\nu=2000\ (m/1eV)$

$$k_{\rm nr} \simeq 0.018\ \Omega_{\rm m}^{1/2} \left(\frac{m}{1\,{\rm eV}}\right)^{1/2} h\,{\rm Mpc}^{-1}$$

Damping of perturbations below this scale
1- Neutrinos cannot be confined into these smaller scales
2- Metric perturbations will be damped at these scales by gravitational back-reaction
3- Modes at k<knr will evolve like in a neutrinoless universe

The perturbed Universe: neutrino free streaming - III

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Taken from Palanque-Delabrouille PONT17 talk

The perturbed Universe: neutrino free streaming - IV

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After decoupling neutrino free-stream

Free streaming scale is a dynamical quantity which quantifies which scales free-streaming can be neglected in the evolution equations at any given time

$$\lambda_{\rm FS} \equiv 2\pi\sqrt{\frac{2}{3}}\frac{c_\nu(\eta)}{H(\eta)}$$

Free streaming horizon is the average distance travelled by neutrinos between the early universe and a given time, displaying the scales that can by affected at allç

$$d_{\rm FS} = a r_{\rm FS} \equiv a\int_{\eta_{in}}^{\eta} c_\nu(\eta) d\eta$$

Comoving horizon scale

Free streaming horizon is the key physical quantity however this role is also taken in the literature by $2\pi/k_{nr}$

NOTE: Neutrinos in the keV range will become non-relativistic in the RD era where $c_\nu \sim 1/a \sim 1/\eta$ and $H \sim \eta^{-2}$ Thus free streaming scale increases like η. While after equality it will decrease. Maximum is in this case reached between equality and non-relativistic transition In this case d_{FS} can be much larger than $1/k_{nr}$ (and grows logarithmically)

Vlasov (neutrino) equation

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$$\frac{Df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = 0$$

1. Perturb f
2. Fill with the metric
3. Go to Fourier

$$\dot{\Psi} - i\frac{q}{\epsilon}(\vec{k}\cdot\hat{n})\Psi = -\left(\dot{\psi} + i\frac{\epsilon}{q}(\vec{k}\cdot\hat{n})\phi\right)\frac{\partial \ln f_0}{\partial \ln q}\,.$$

Neutrino phase space

Metric

Vlasov (neutrino) equation in MD regime

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$$\frac{dq}{d\tau} = q\dot{\psi} + (q^2 + a^2 m^2)^{1/2} \hat{n}_i \, \partial_i \phi$$

Geodesic equation

$$\Psi(\vec{k}, q, \hat{n}, \tau) = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l(\vec{k}, q, \tau) P_l(\hat{k} \cdot \hat{n}) \, .$$

Then, the Vlasov equation becomes

$$\dot{\Psi}_0 = \frac{q\,k}{\epsilon} \Psi_1 - \dot{\psi} \frac{\partial \ln f_0}{\partial \ln q} \, ,$$

$$\dot{\Psi}_1 = \frac{q\,k}{3\,\epsilon} (\Psi_2 - 2\Psi_0) + \frac{\epsilon\,k}{3\,q} \phi \frac{\partial \ln f_0}{\partial \ln q} \, ,$$

$$\dot{\Psi}_l = \frac{q\,k}{(2l+1)\,\epsilon} \left[(l+1)\Psi_{l+1} - l\Psi_{l-1} \right] \, , \quad l \geq 2 \, .$$

$$\delta\rho_\nu = 4\pi a^{-4} \int q^2 dq \, \epsilon f_0(q) \Psi_0 \, ,$$

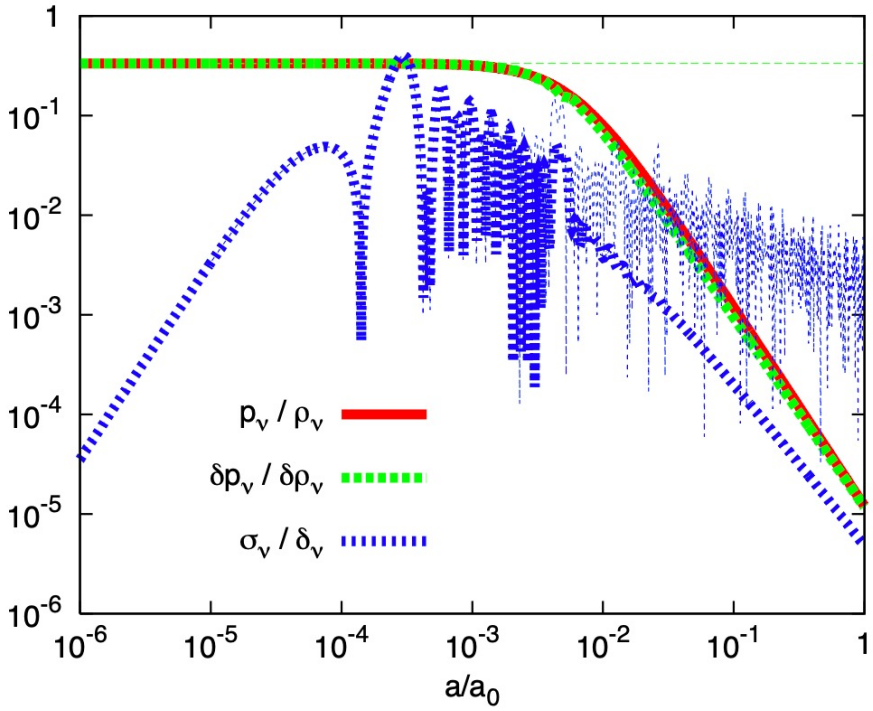
$$\delta p_\nu = \frac{4\pi}{3} a^{-4} \int q^2 dq \, \frac{q^2}{\epsilon} f_0(q) \Psi_0 \, ,$$

$$(\bar{\rho}_\nu + \bar{p}_\nu)\theta_\nu = 4\pi k a^{-4} \int q^2 dq \, q f_0(q) \Psi_1 \, ,$$

$$(\bar{\rho}_\nu + \bar{p}_\nu)\sigma_\nu = -\frac{8\pi}{3} a^{-4} \int q^2 dq \, \frac{q^2}{\epsilon} f_0(q) \Psi_2 \, .$$

Vlasov (neutrino) equation in MD regime - II

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This is for the mode k=0.1 h/Mpc
And for m=0.1 eV

Damping is clearly visible
After non-relativistic transition
$a/a_0 = 5 \times 10^{-3}$

Vlasov (neutrino) equation in MD regime - III

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Well in the matter dominated regime, things get simpler

$$\dot{\delta}_\nu = \theta_\nu + 3\dot{\psi} \ ,$$
$$\dot{\theta}_\nu = -\frac{\dot{a}}{a}\theta_\nu - k^2\phi \ .$$

Psi and phi ~ const and a ~ τ^2

$$\ddot{\delta}_\nu + \frac{\dot{a}}{a}\dot{\delta}_\nu = -k^2\phi + 3(\ddot{\psi} + \frac{\dot{a}}{a}\dot{\psi})$$

$$\delta_\nu = A\ \ln\tau + B - \frac{(k\tau)^2}{6}\phi$$
$$= \tilde{A}\ \ln a + \tilde{B} - \frac{2}{3}\left(\frac{k}{aH}\right)^2\phi \ ,$$

$$\delta_\nu \longrightarrow -\frac{2}{3}\left(\frac{k}{aH}\right)^2\phi \ \propto a \ ;$$

This above is solution of Poisson Equation in a MD Universe
For $k>k_{nr}$
Note that δ_ν can grow faster than ~ a
For a short time due to the ln a term

Vlasov (neutrino) equation in MD regime - III

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From the above equations and neglecting neutrinos' backreaction on cdm

$$
\begin{aligned}
P(k) &= \left\langle \left(\frac{\delta\rho_{\rm cdm} + \delta\rho_{\rm b} + \delta\rho_{\nu}}{\rho_{\rm cdm} + \rho_{\rm b} + \rho_{\nu}} \right)^2 \right\rangle \\
&= \left\langle \left(\frac{\Omega_{\rm cdm}\,\delta_{\rm cdm} + \Omega_{\rm b}\,\delta_{\rm b} + \Omega_{\nu}\,\delta_{\nu}}{\Omega_{\rm cdm} + \Omega_{\rm b} + \Omega_{\nu}} \right)^2 \right\rangle \\
&= \begin{cases} \langle \delta^2_{\rm cdm} \rangle & \text{for} \quad k < k_{\rm nr}, \\ [1 - \Omega_{\nu}/\Omega_{\rm m}]^2 \langle \delta^2_{\rm cdm} \rangle & \text{for} \quad k \gg k_{\rm nr}, \end{cases}
\end{aligned}
$$

From the above equations and neglecting neutrinos' backreaction on cdm

Vlasov (neutrino) equation in MD regime - III

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Well in the matter dominated regime, things get simpler

$$\dot{\delta}_\nu = \theta_\nu + 3\dot{\psi} \ ,$$
$$\dot{\theta}_\nu = -\frac{\dot{a}}{a}\theta_\nu - k^2\phi \ .$$

Psi and phi ~ const and a ~ τ^2

$$\ddot{\delta}_\nu + \frac{\dot{a}}{a}\dot{\delta}_\nu = -k^2\phi + 3(\ddot{\psi} + \frac{\dot{a}}{a}\dot{\psi})$$

$$\delta_\nu = A\ \ln\tau + B - \frac{(k\tau)^2}{6}\phi$$
$$= \tilde{A}\ \ln a + \tilde{B} - \frac{2}{3}\left(\frac{k}{aH}\right)^2\phi \ ,$$

$$\delta_\nu \longrightarrow -\frac{2}{3}\left(\frac{k}{aH}\right)^2\phi \ \propto a \ ;$$

This above is solution of Poisson Equation in a MD Universe
For k>knr
Note that δν can grow faster than ~ a
For a short time due to the ln a term

Neutrinos backreactions

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1. Effects from homogenous pressure and density → Hubble expansion
2. Gravitational back-reaction on metric perturbations through modification of energy-momentum tensor

To check for 1. we can omit the d n in Poission equation

$$\ddot{\delta}_{\rm cdm} + \frac{\dot{a}}{a}\dot{\delta}_{\rm cdm} = -k^2\phi + 3(\ddot{\psi} + \frac{\dot{a}}{a}\dot{\psi})$$

$$\ddot{\delta}_{\rm cdm} + \frac{2}{\tau}\dot{\delta}_{\rm cdm} - \frac{6}{\tau^2}\delta_{\rm cdm} = 0 \ ,$$

$$f_\nu \equiv \frac{\rho_\nu}{(\rho_{\rm cdm}+\rho_{\rm b}+\rho_\nu)} = \frac{\Omega_\nu}{\Omega_{\rm m}}$$

$$\ddot{\delta}_{\rm cdm} + \frac{2}{\tau}\dot{\delta}_{\rm cdm} - \frac{6}{\tau^2}(1-f_\nu)\,\delta_{\rm cdm} = 0$$

$$\delta_{\rm cdm} \propto \tau^{2p}$$

$$\delta_{\rm cdm} \propto a^{p+} \simeq a^{1-\frac{3}{5}f_\nu}$$

$$-k^2\psi \propto a^{p+-1} \simeq a^{-\frac{3}{5}f_\nu}$$ Poisson eq.

$$\delta_{\rm cdm} \propto [a\,g(a)]^{p+} \simeq [a\,g(a)]^{1-\frac{3}{5}f_\nu} \ .$$

Matter power spectrum from massive vs massless vs

Matteo Viel

1. Large scales k < knr

2. k>> knr and k >> keq for a< anr $\delta^{f_\nu}_{\rm cdm}[a] = \delta^{f_\nu=0}_{\rm cdm}[(1-f_\nu)\,a]$ $a^{f_\nu}_{\rm eq}/a^{f_\nu=0}_{\rm eq} = (1-f_\nu)^{-1}$

$$\delta^{f_\nu}_{\rm cdm}[a_0] = \left(\frac{a_0\, g(a_0)}{a_{\rm nr}}\right)^{1-\frac{3}{5}f_\nu} \delta^{f_\nu}_{\rm cdm}[a_{\rm nr}]$$

$$\frac{\delta^{f_\nu}_{\rm cdm}[a_0]}{\delta^{f_\nu=0}_{\rm cdm}[a_0]} = (1-f_\nu)^{1/2}\left(\frac{a_0\, g(a_0)}{a_{\rm nr}}\right)^{-\frac{3}{5}f_\nu}$$

$$\frac{P(k)^{f_\nu}}{P(k)^{f_\nu=0}} = (1-f_\nu)^3\left(\frac{a_0\, g(a_0)}{a_{\rm nr}}\right)^{-\frac{6}{5}f_\nu} = (1-f_\nu)^3\left[1.9\times10^5\, g(a_0)\,\omega_{\rm m}\, f_\nu/N_\nu\right]^{-\frac{6}{5}f_\nu}$$

$$\frac{P(k)^{f_\nu}}{P(k)^{f_\nu=0}} \simeq -8\, f_\nu\ .$$

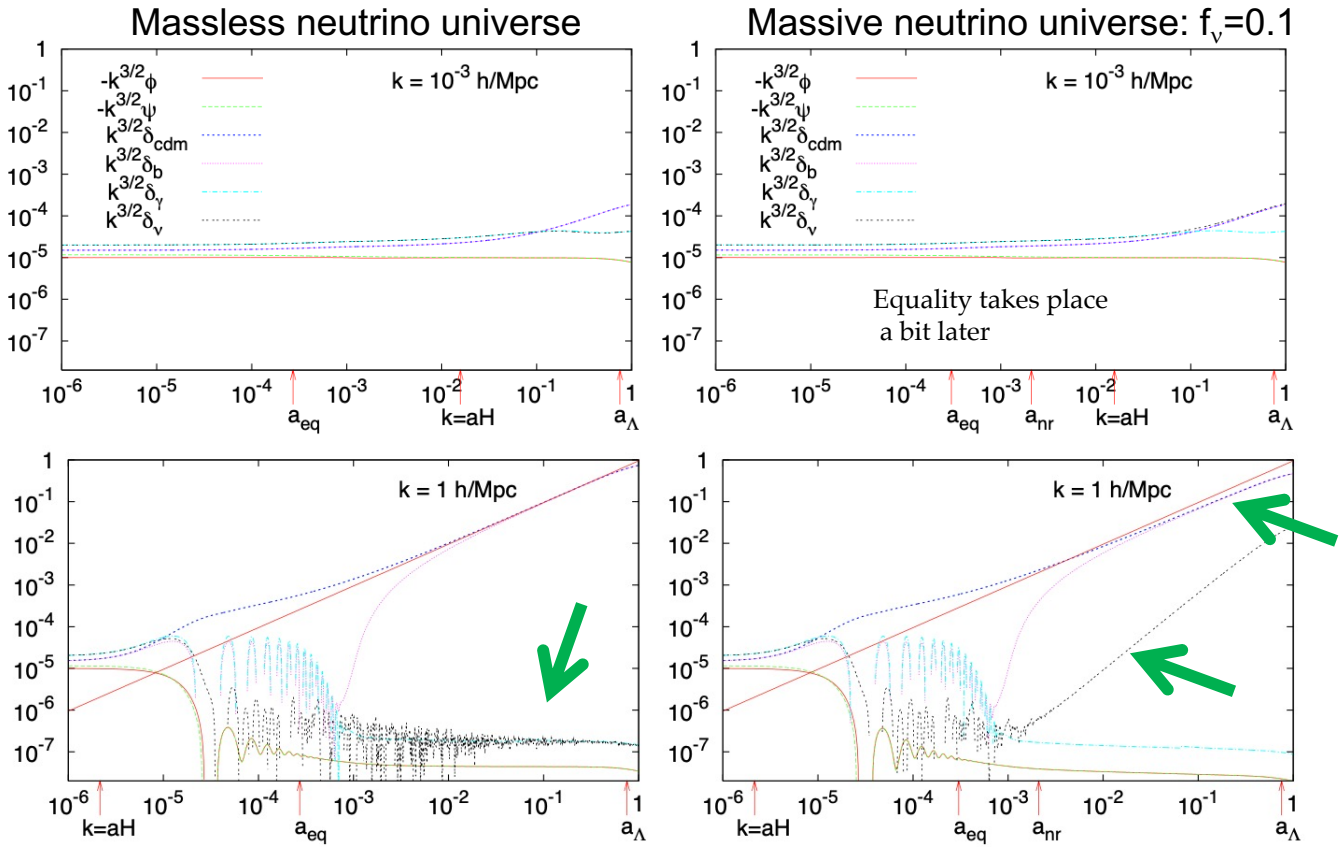
Small few % effect? Yes!
But "integrated" through structure formation era i.e. P(k) is P(k,z)

But… is it really small ---> see Enzo Branchini!

Matter power spectrum from massive vs massless vs

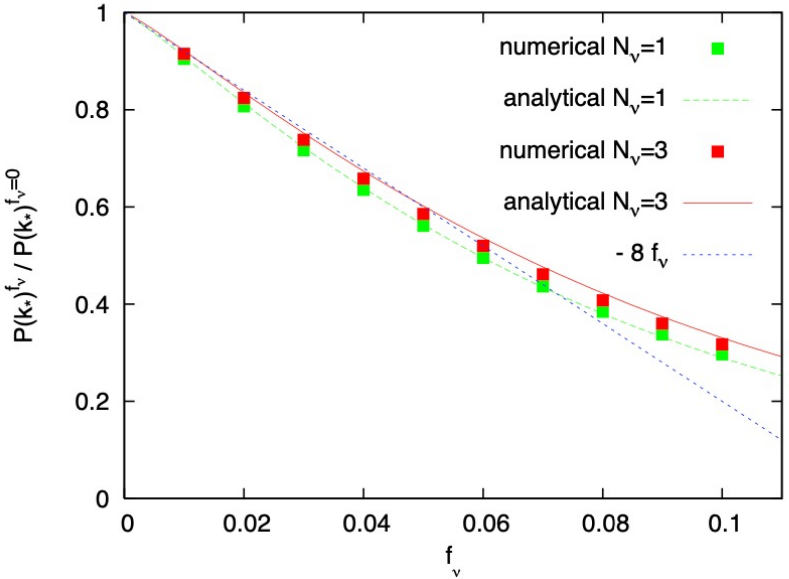
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In practice: numerical solutions → popular Boltzmann solvers like CMBFAST and CLASS



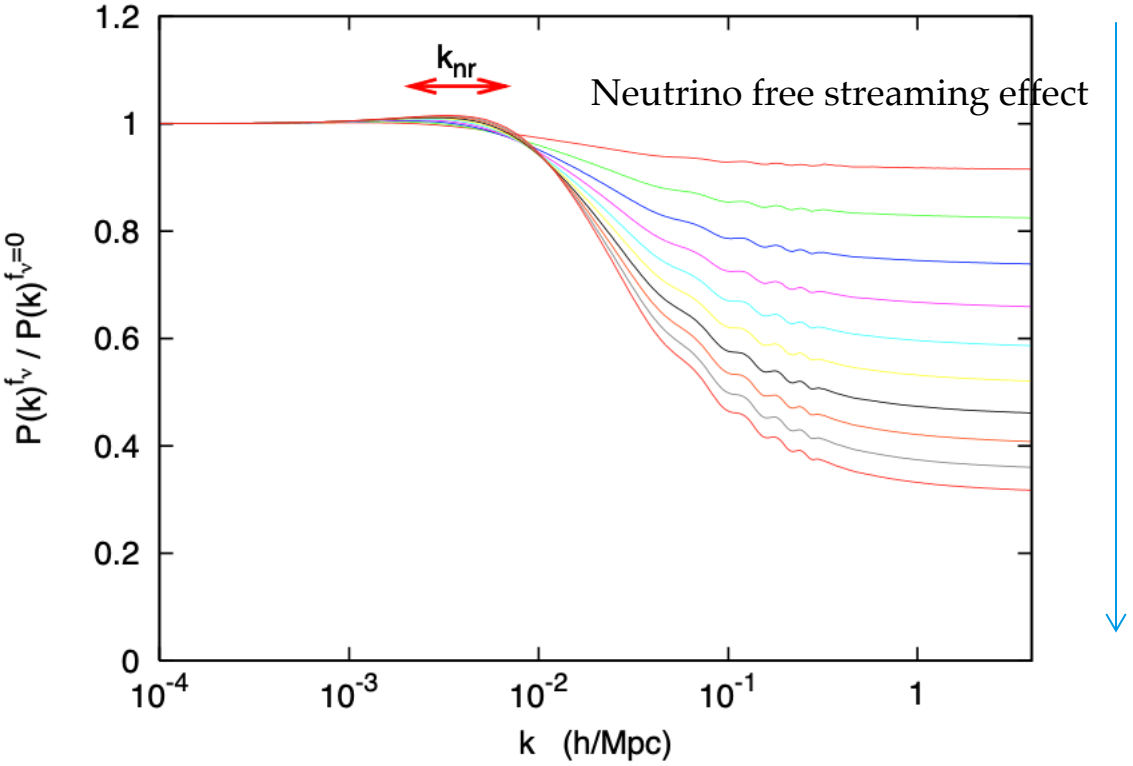
Matter power spectrum from massive vs massless vs

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Matter power spectrum from massive vs massless vs

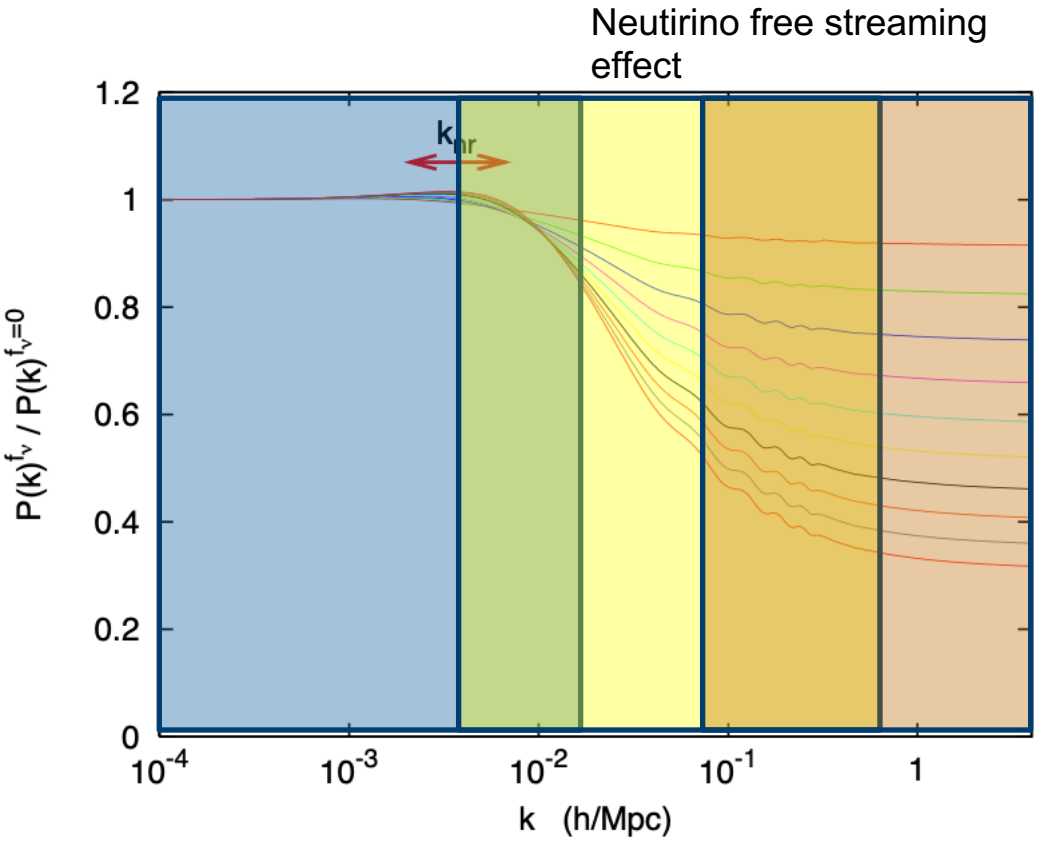
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From 0.15 eV to 1.5eV total neutrino mass

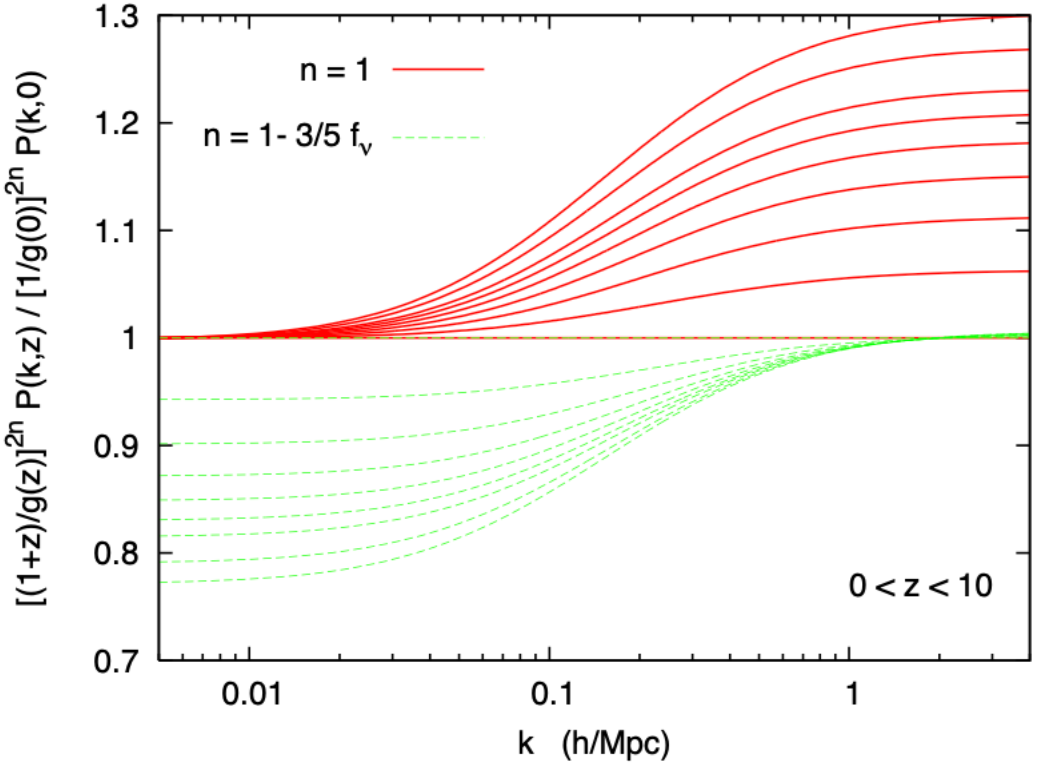
Matter power spectrum from massive vs massless vs

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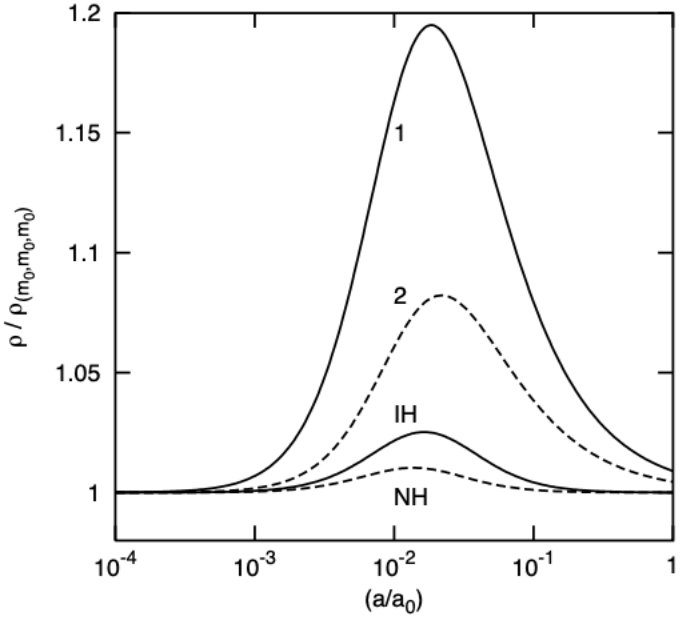
Matter power spectrum from massive vs massless vs

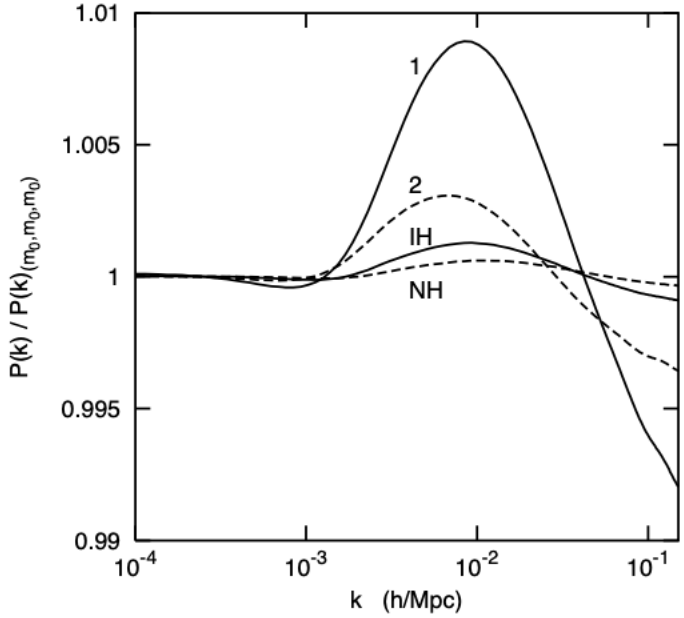
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Matter power spectrum from massive vs massless vs

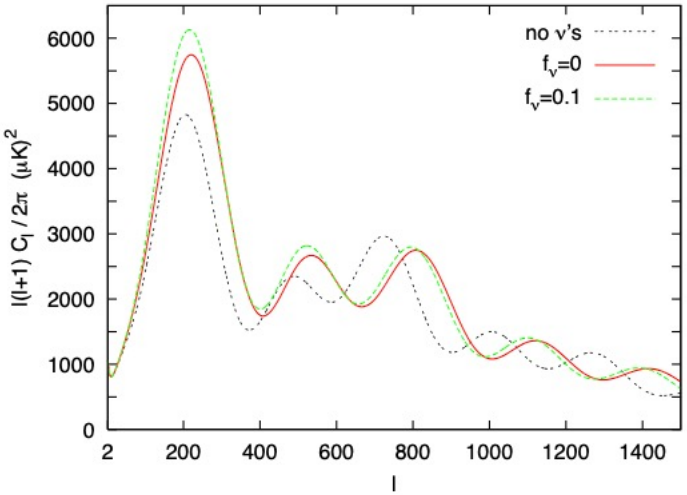
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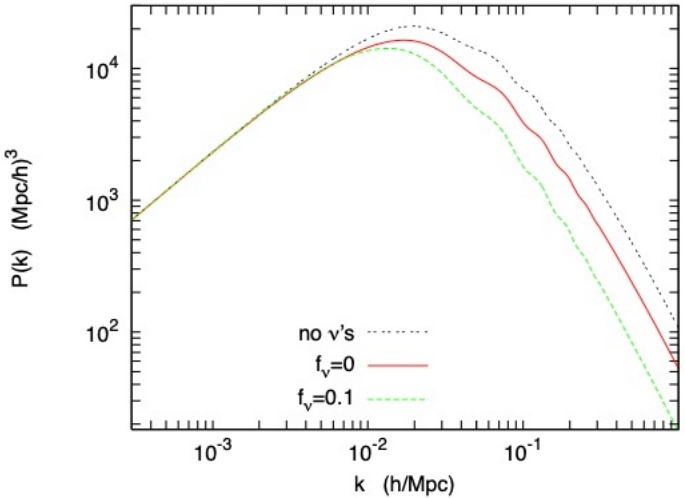




Matter power spectrum from massive vs massless vs

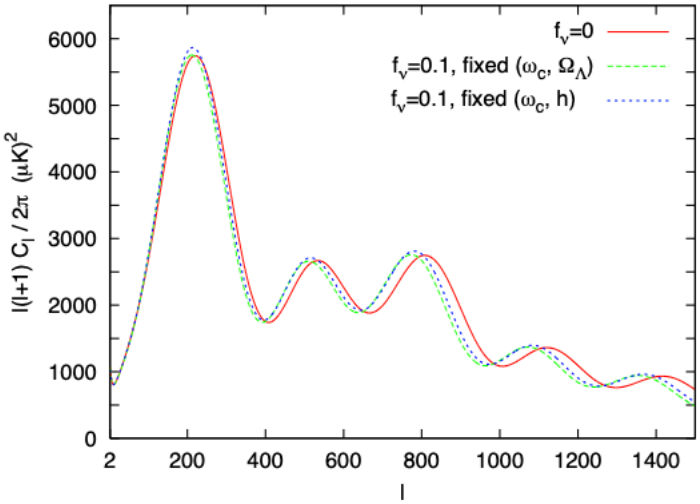
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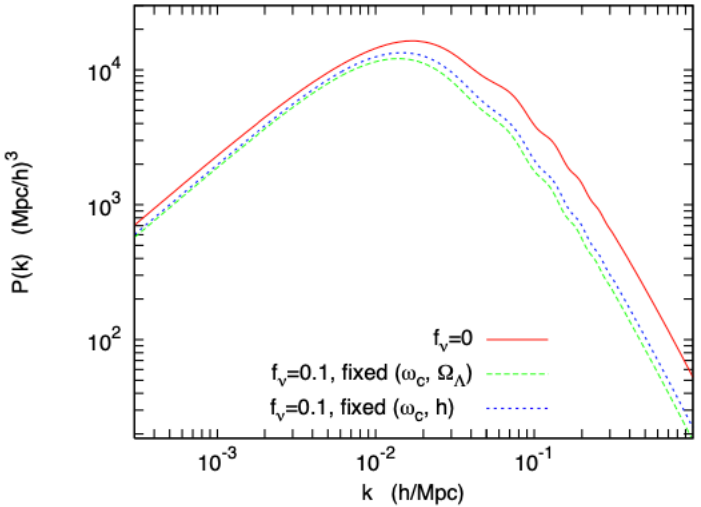




Matter power spectrum from massive vs massless vs

Matteo Viel





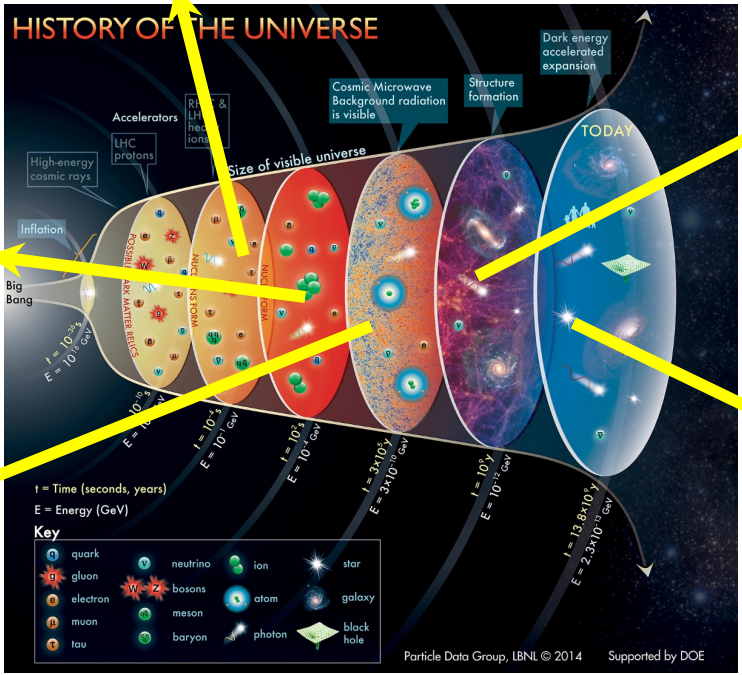
Recap - key moments

Matteo Viel

1. Relativistic neutrino contribution to cosmic expansion

2. Neutrino free streaming slows down CMB photon clustering

3. Metric fluctuations During non-relativistic neutrino transition (early ISW)



4. Neutrino free streaming slows down ordinary matter clustering

5. Non-relativistic neutrino contribution to late expansion rate

BBN CMB LSS

Recap - key take home messages

Matteo Viel

1. Neutrino number density is large, like photons, in terms of number density is the second most abundant species
   $n_{\gamma} \sim n_{\nu} \sim 10^{10} n_{\text{atoms, e-}}$

2. Unlike other particles they become non-relativistic **after** decoupling and do not annihilate

3. Looking at the whole Universe from large to small scales they can be probed

Plan

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Regimes

$\delta \ll 1$ **Linear theory**: use public codes like CAMB/CLASS or your own solver
analytical neutrino calculations also valid in some regimes

$\delta \leq 1$ **Perturbation theories**: Standard Perturbation theory or other methods, Effective field theory of the LSS

any δ **Non-linear regime: A)** analytical models for non-linear power like the halo model (or some log-normal approximation for the density field)
B) N-body simulations

N-body sims in the context of neutrino cosmologies

Matteo Viel

1980: Bond et al. 1980 – linear theory (also Russian school with Zeldovich)
1983: Bond et al. – Evolution of Boltzmann-Einstein equations. Clustering properties of galaxies not reproduced if the universe is dominated by neutrinos (White et al. 1983) – numerical experiment

1992: Davis et al. HDM or CHDM models P3M codes with neutrino particles placed as the dark matter ones (same CDM spectrum + velocities): 32^3 particles

1993: Klypin et al. 2 x 128^3 particles at z_IC=14 with the right power spectrum

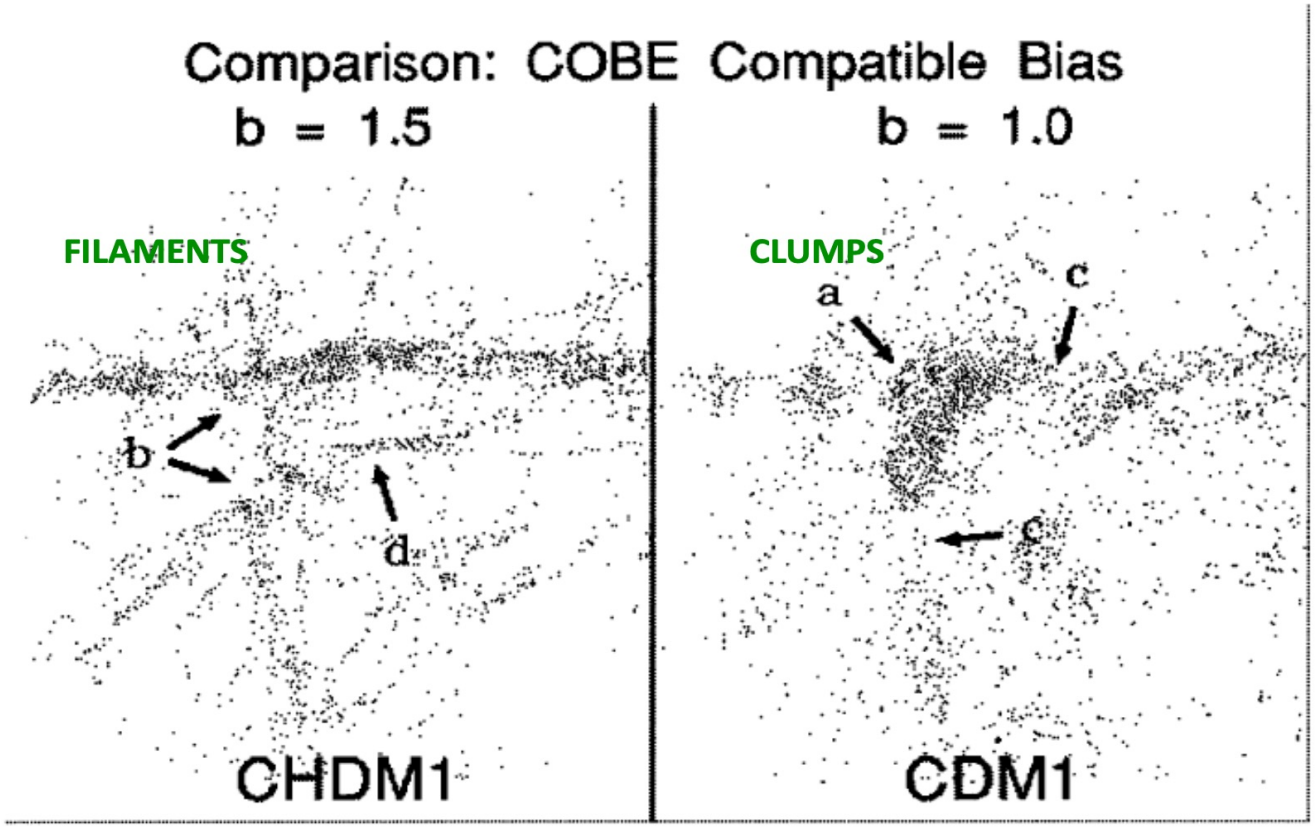
1994: Ma & Bertschinger approximate linear scheme evolved at z=13 and after that pure N-body



N-body sims in the context of neutrino cosmologies

Matteo Viel

Pure HDM not allowed. However CHDM is still viable and impacts on the cosmic web

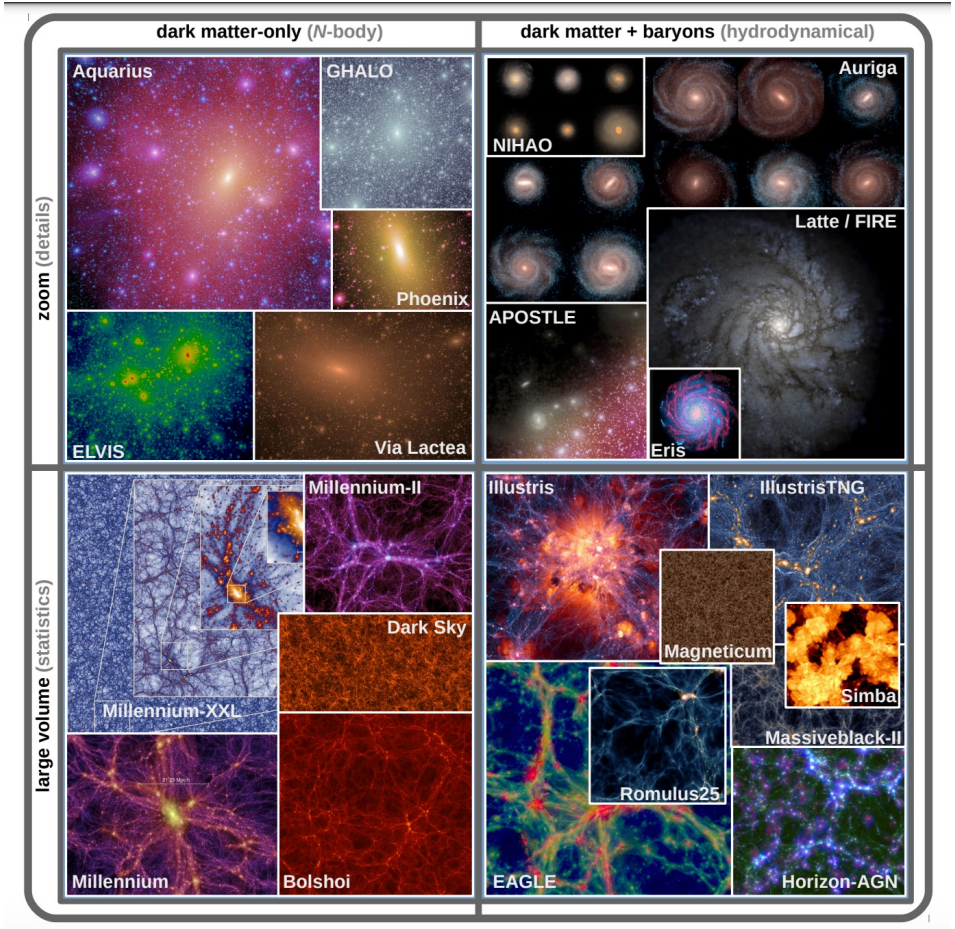


Brodbeck+98

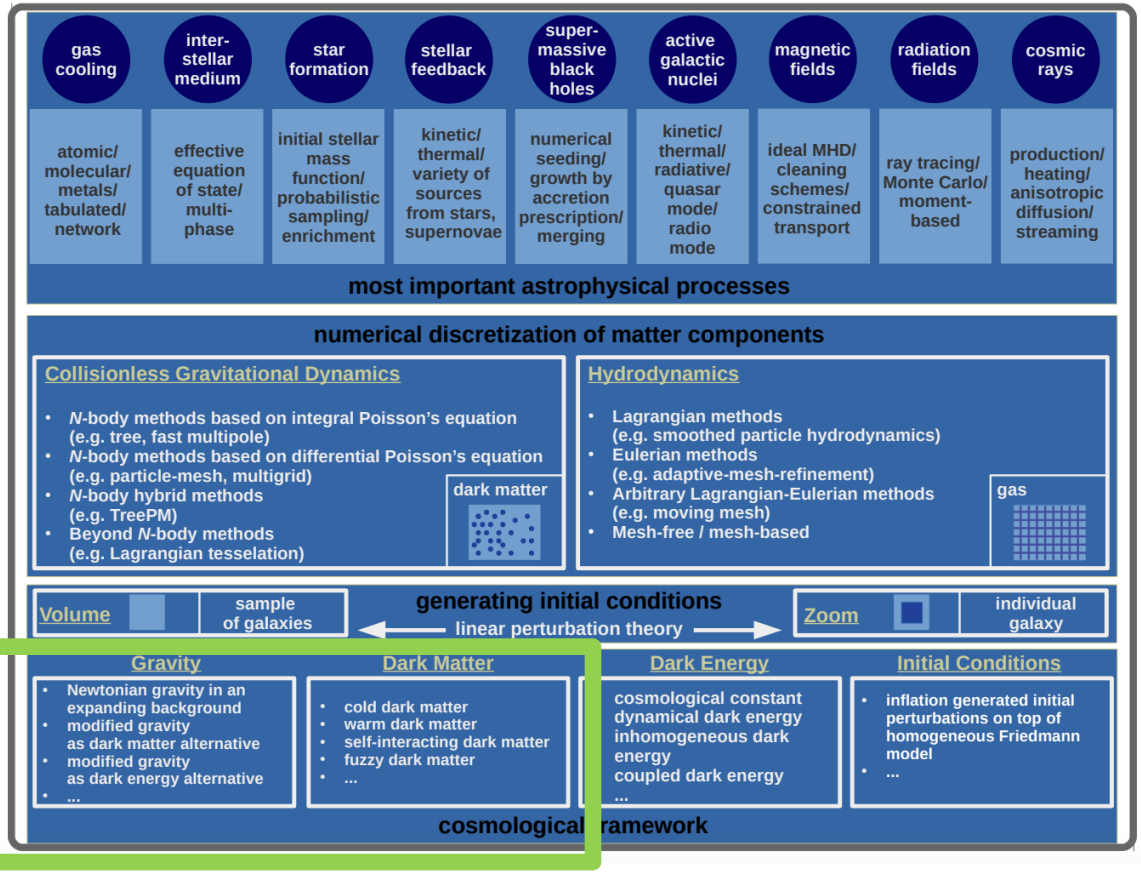
Numerical methods for structure formations

Matteo Viel

Vogelsberger+20
Nature Review



Physics implemented



Setting Initial conditions

Matteo Viel

It is wise to start the simulations in the MD era, to set up initial conditions we will use the Results of Lecutre 1: i.e. linear theory

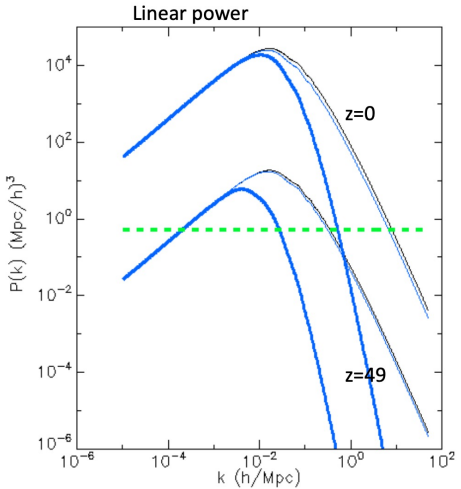
Generating initial conditions

initial positions: $\mathbf{x} = \mathbf{q} + D(t)\Psi(\mathbf{q})$ **initial velocities:** $a(t)\dot{\mathbf{x}} = a(t)\frac{\mathrm{d}D(t)}{\mathrm{d}t}\Psi(\mathbf{q}) = a(t)H(t)\frac{\mathrm{d}\ln D}{\mathrm{d}\ln a}D(t)\Psi(\mathbf{q})$

Comoving initial positions, $\mathbf{x}$, are assigned based on the unperturbed particle position, $\mathbf{q}$, the linear growth factor, $D(t)$, and the scale factor, a, which is related to the initial redshift, $z = 1/a - 1$. The curl-free displacement field Ψ is computed by solving the linearized continuity equation $\nabla \cdot \Psi = -\delta/D(t)$, where δ is the relative density fluctuation.

Zel'dovich approximation is used

However: note that for neutrinos
the initial FD thermal velocities are large
Note also: Poissonian shot noise



Black is matter power

Blue is matter power in neutrino simulations

Thick blue is neutrino power

Green dashed is shot noise level for a typical run

Zeldovich displacement of the particles

$$\boldsymbol{x}(\boldsymbol{q}, z) = \boldsymbol{q} + D_+(z)\nabla_q \phi_{\mathrm{i}}(\boldsymbol{q})$$

$$\phi_{\mathrm{i}}(\boldsymbol{x}) = -\frac{3}{2}H^2 a^3 \Phi_{\mathrm{i}}(\boldsymbol{x})$$

Setting ICs: Zel'dovich approximation

Matteo Viel

Particle displacements:

$$\mathbf{d}_i(t) = \mathbf{x}_i(t) - \mathbf{q}_i$$

Density change due to displacements:

$$\rho(\mathbf{x}) = \frac{\rho_0}{\left|\frac{\partial \mathbf{x}}{\partial \mathbf{q}}\right|} = \frac{\rho_0}{\left|\delta_{ij} + \frac{\partial \mathbf{d}}{\partial \mathbf{q}}\right|}$$

For small displacements:

$$\left|\delta_{ij} + \frac{\partial \mathbf{d}}{\partial \mathbf{q}}\right| \simeq 1 + \nabla_{\mathbf{q}} \cdot \mathbf{d}$$

Resulting density contrast:

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_0}{\rho} = -\nabla_{\mathbf{q}} \cdot \mathbf{d}$$

During linear growth:

$$\delta(t) = D(t)\delta_0$$

$$\mathbf{d}(t) = D(t)\mathbf{d}_0$$

$$\Rightarrow \quad \dot{\mathbf{x}} = \dot{\mathbf{d}} = \dot{a}\frac{\mathrm{d}D}{\mathrm{d}a}\mathbf{d}_0 = \frac{\dot{a}}{a}\frac{a}{D}\frac{\mathrm{d}D}{\mathrm{d}a}\mathbf{d}$$

Particle velocities:

$$\dot{\mathbf{x}} = H(a)f(\Omega)\mathbf{d} \qquad f(\Omega) = \frac{\mathrm{d}\ln D}{\mathrm{d}\ln a} \simeq \Omega^{0.6}$$

Note: Particles move on straight lines in the Zeldovich approximation.

N-body sims primer

Matteo Viel

Modeling dark matter

collisionless Boltzmann equation: $\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0$ **Poisson's equation:** $\nabla^2\Phi = 4\pi G \int f \mathrm{d}\mathbf{v}$

The collisionless Boltzmann equation describes the evolution of the phase-space density or distribution function of dark matter, $f = f(\mathbf{r},\mathbf{v},t)$, under the influence of the collective gravitational potential, Φ, given by Poisson's equation. The collisionless Boltzmann equation states the conservation of the local phase-space density; i.e. Liouville's theorem.

Table 1: Major galaxy formation simulation codes

code name	gravity treatment[a]	hydrodynamics treatment[b]	parallelization technique[c]	code availability[d]	primary reference
ART	PM/ML	AMR	data-based	public	Kravtsov (1997)[27]
RAMSES	PM/ML	AMR	data-based	public	Teyssier (2002)[38]
GADGET-2/3	TreePM	SPH	data-based	public	Springel (2005)[39]
Arepo	TreePM	MMFV	data-based	public	Springel (2010)[40]
Enzo	PM/MG	AMR	data-based	public	Bryan et al. (2014)[41]
ChaNGa[e]	Tree/FM	SPH	task-based	public	Menon et al. (2015)[42–44]
GIZMO[f]	TreePM	MLFM/MLFV	data-based	public	Hopkins et al. (2015)[45]
HACC	TreePM/P^3M	CRK-SPH	data-based	private	Habib et al. (2016)[46]
PKDGRAV3	Tree/FM	–	data-based	public	Potter et al. (2017)[47]
Gasoline2	Tree	SPH	task-based	public	Wadsley et al. (2017)[48]
SWIFT	TreePM/FM	SPH	task-based	public	Schaller et al. (2018)[49]

[a] PM: particle-mesh; TreePM: tree + PM, FM: fast multipole, P^3M: particle-particle-particle-mesh; ML: multilevel; MG: multigrid
[b] SPH: smoothed particle hydrodynamics, CRK-SPH: conservative reproducing kernel smoothed particle hydrodynamics , AMR: adaptive-mesh-refinement, MMFV: moving-mesh finite volume, MLFM/MLFV: mesh-free finite mass / finite volume
[c] data-based: data parallelism focuses on distributing data across different nodes, which operate on the data in parallel; task-based: task parallelism focuses on distributing tasks concurrently performed
[d] private: private code; public: publicly available code (in some cases with limited functionality)
[e] gravity solver is based on PKDGRAV3
[f] based on the GADGET-3 code

N-body sims: the Particle Mesh method

Matteo Viel

$$\ddot{\mathbf{x}}_i = -\nabla_i \Phi(\mathbf{x}_i)$$

$$\Phi(\mathbf{x}) = -G \sum_{j=1}^{N} \frac{m_j}{[(\mathbf{x}-\mathbf{x}_j)^2 + \epsilon^2]^{1/2}}$$

Note: The naïve computation of the forces is an N^2 - task.

Direct summation is unfeasible
One can rely on approximations

Poisson's equation can be solved in real-space by a convolution of the density field with a Green's function.

$$\Phi(\mathbf{x}) = \int g(\mathbf{x}-\mathbf{x}')\,\rho(\mathbf{x})\,\mathrm{d}\mathbf{x}'$$

In Fourier-space, the convolution becomes a simple multiplication!

$$\hat{\Phi}(\mathbf{k}) = \hat{g}(\mathbf{k}) \cdot \hat{\rho}(\mathbf{k})$$

→ **Solve the potential in these steps:**

(1) FFT forward of the density field
(2) Multiplication with the Green's function
(3) FFT backwards to obtain potential

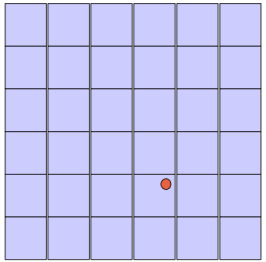
The four steps of the PM algorithm
(a) Density assignment
(b) Computation of the potential
(c) Determination of the force field
(d) Assignment of forces to particles

N-body sims: force calculation in the PM method

Matteo Viel

Finite differencing of the potential to get the force field

Approximate the force field $\mathbf{f} = -\nabla\Phi$ with finite differencing



2nd order accurate scheme:

$$f^{(x)}_{i,j,k} = -\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h}$$

4th order accurate scheme:

$$f^{(x)}_{i,j,k} = -\frac{4}{3}\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h} + \frac{1}{3}\frac{\Phi_{i+2,j,k} - \Phi_{i-2,j,k}}{4h}$$

Interpolating the mesh-forces to the particle locations

$$F(\mathbf{x}_i) = \sum_{\mathbf{m}} W(\mathbf{x}_i - \mathbf{x}_\mathbf{m})\, f_\mathbf{m}$$

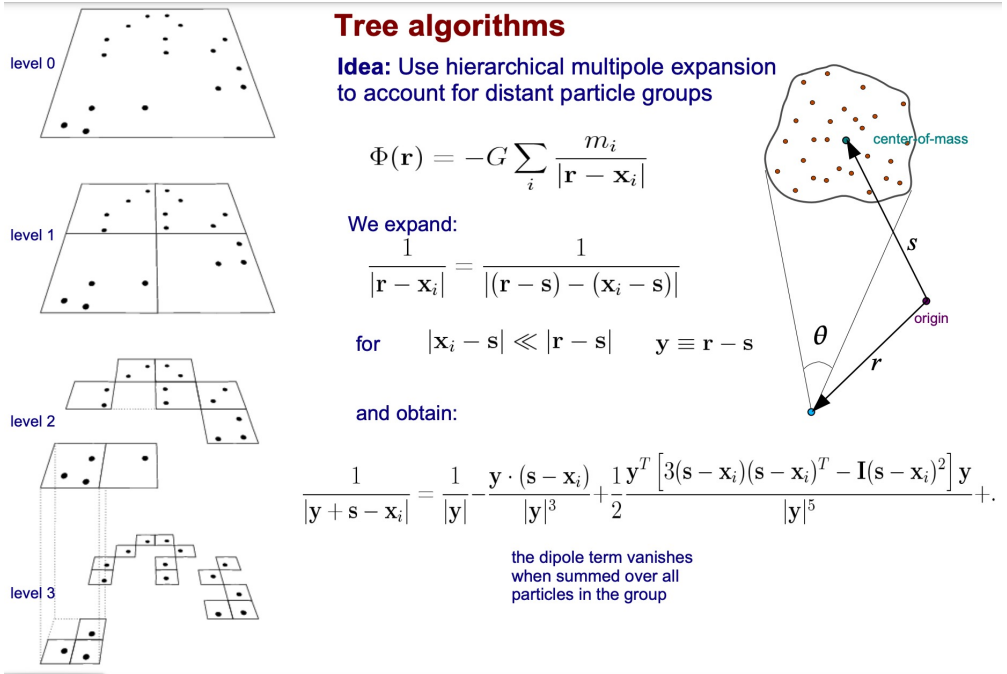
The interpolation kernel needs to be the same one used for mass-assignment to ensure force anti-symmetry.

N-body sims: method

Matteo Viel

PM methods are fast and simple they are good for species which do not cluster much (neutrinos?)
However, DM will cluster more, PM is not good (limited by force resolution on the mesh).

Tree algorithm: group distant particles together and use their multipole expansion only log(N) force Terms per particle



N-body sims: Tree-PM

Matteo Viel

THE TREE-PM FORCE SPLIT

Periodic peculiar potential

$$\nabla^2\phi(\mathbf{x}) = 4\pi G[\rho(\mathbf{x}) - \overline{\rho}] = 4\pi G \sum_{\mathbf{n}} \sum_{i} m_i \left[\tilde{\delta}(\mathbf{x} - \mathbf{x}_i - \mathbf{n}L) - \frac{1}{L^3}\right]$$

Idea: Split the potential (of a single particle) in Fourier space into a long-range and a short-range part, and compute them separately with PM and TREE algorithms, respectively.

Poisson equation in Fourier space:

$$\phi_{\mathbf{k}} = -\frac{4\pi G}{\mathbf{k}^2}\rho_{\mathbf{k}} \quad (\mathbf{k} \neq 0)$$

DM small scale: Tree
DM large scale: PM
Neutrinos (all over): PM

$$\phi_{\mathbf{k}}^{\rm long} = \phi_{\mathbf{k}} \exp(-\mathbf{k}^2 r_s^2)$$

Solve with PM-method

- CIC mass assignment
- FFT
- multiply with kernel
- FFT backwards
- Compute force with 4-point finite difference operator
- Interpolate forces to particle positions

$$\phi_{\mathbf{k}}^{\rm short} = \phi_{\mathbf{k}} \left[1 - \exp(-\mathbf{k}^2 r_s^2)\right]$$

FFT to real space

$$\phi(r) = -\frac{Gm}{r}\,\mathrm{erfc}\left(\frac{r}{2r_s}\right)$$

Solve in real space with TREE

Key results of N-body simulations

1) Large/Medium/small scale distribution of matter

2) Halo mass function

3) Structure of dark matter haloes

4) Halo substructure

Till 2008 all the above points were tackled without considering neutrinos as a Separate fluid but implementing the matter power spectrum of a massive neutrino cosmology

Key results of N-body simulations

1994A

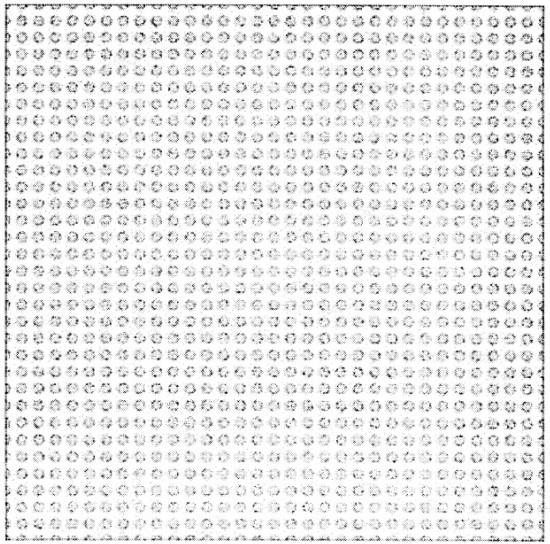
A CALCULATION OF THE FULL NEUTRINO PHASE SPACE IN COLD + HOT DARK MATTER MODELS

CHUNG-PEI MA[1] AND EDMUND BERTSCHINGER
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

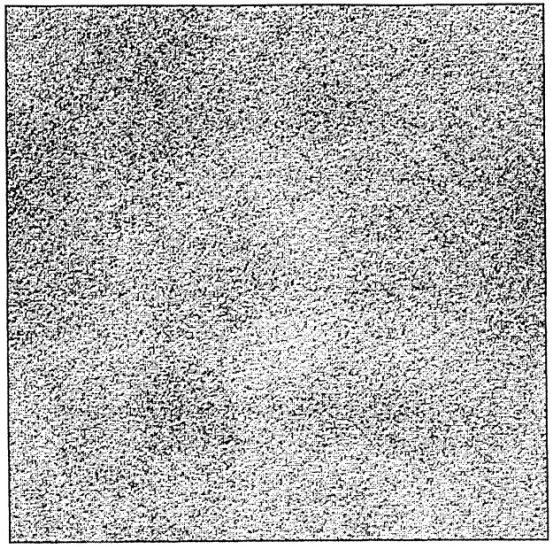
Received 1993 August 6; accepted 1993 December 30

Monte Carlo method to sample (integrate geodesic equation) the neutrino phase space-density from $z=10^9$ to $z=0$ First they use linear theory to z=13, then they switch to N-body

z=10,000

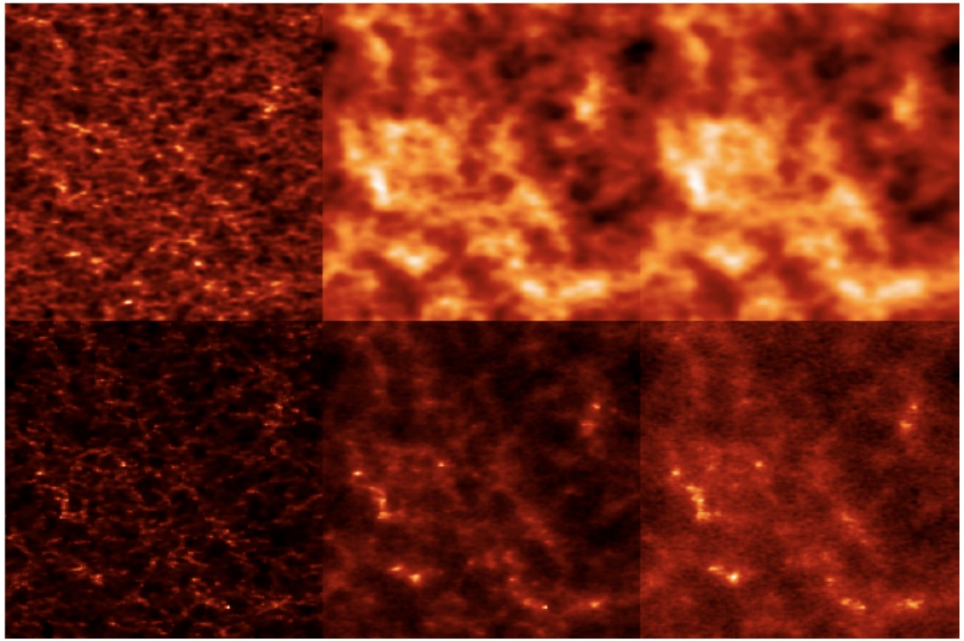


z=13.5



Neutrino simulation with particles

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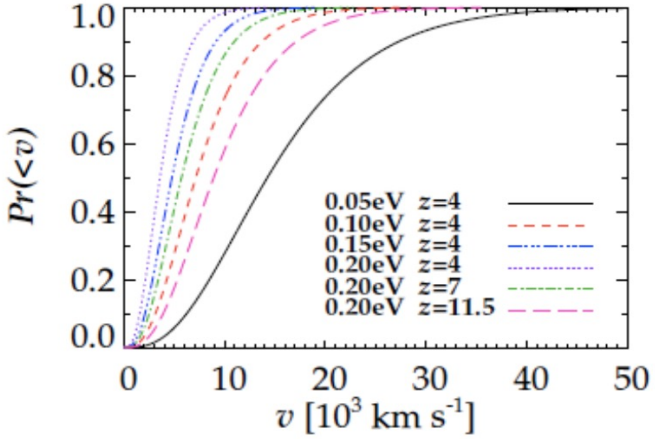
Simulation of neutrinos as an independent set of particles that interact gravitationally

COLD DM NEUTRINOS 0.6 eV NEUTRINOS 0.3 eV

Brandbyge et al 08

Neutrino simulations: role of thermal velocities

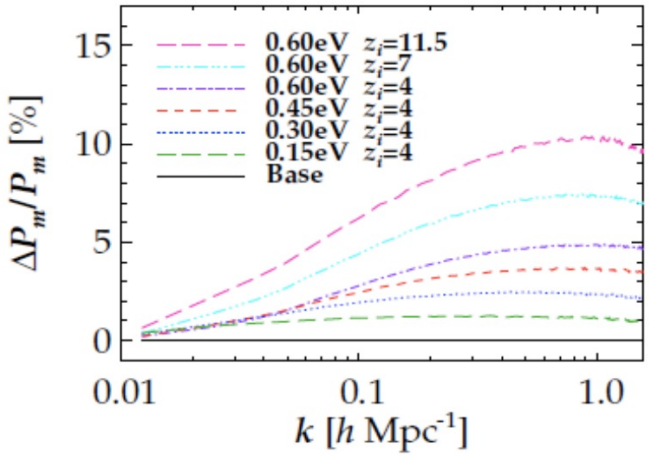
Matteo Viel



$$T_\nu \simeq T_\gamma (4/11)^{1/3}$$

$$Pr(<p) = N \int_0^p \frac{p'^2}{e^{p'c/k_b T_\nu} + 1} dp'$$

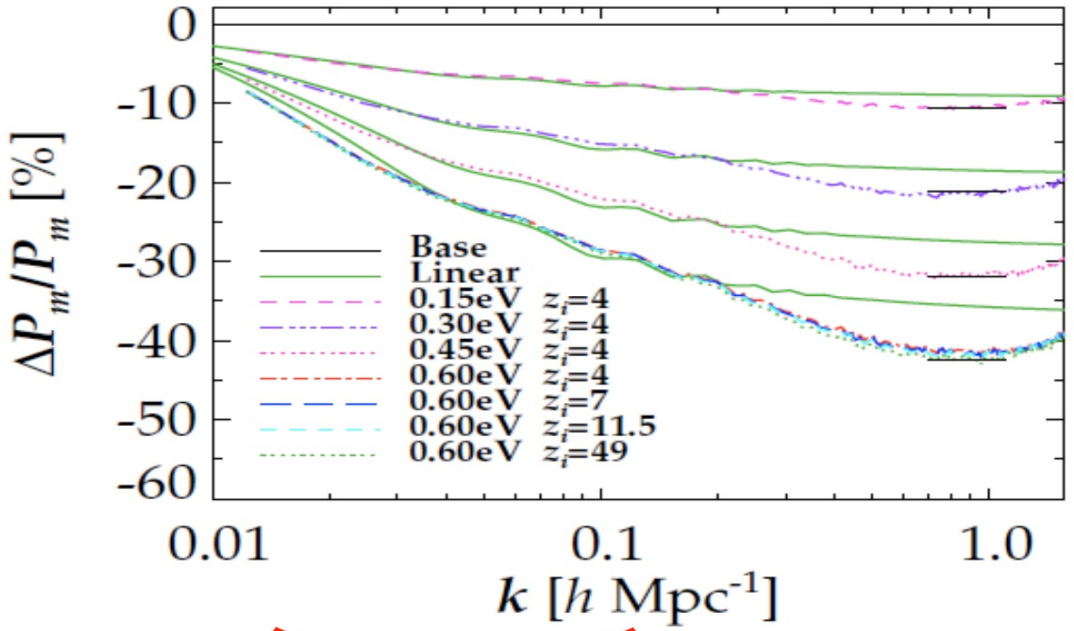
Draw velocity from Fermi-Dirac distribution



Brandbyge et al 08a

Non-linear matter power

Matteo Viel



Brandbyge et al 08

$$\frac{\Delta P}{P}\Big|_{\max} \sim -8\frac{\Omega_\nu}{\Omega_m} \rightarrow \frac{\Delta P}{P}\Big|_{\max} \sim -9.8\frac{\Omega_\nu}{\Omega_m}$$

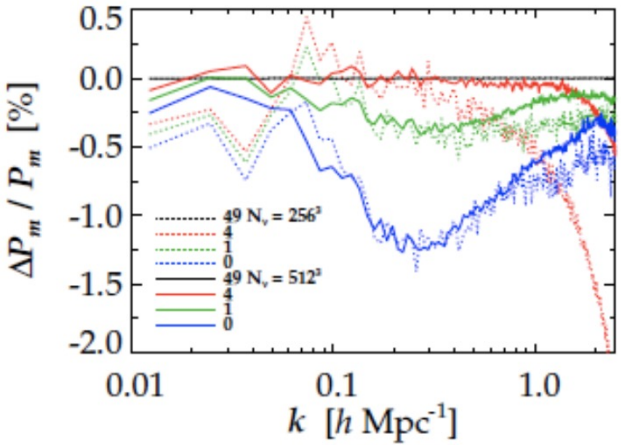
Non-linear matter power

Matteo Viel

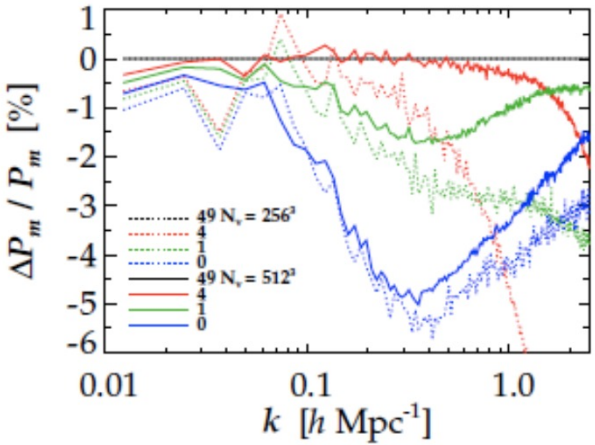
Computing the neutrino gravitational potential on the PM grid and summing up its contribution to the total matter gravitational potential

COMPARISON GRID VS PARTICLES

M ν =0.6 eV



M ν = 1.2 eV



Brandbyge et al 08b

Non-linear matter power

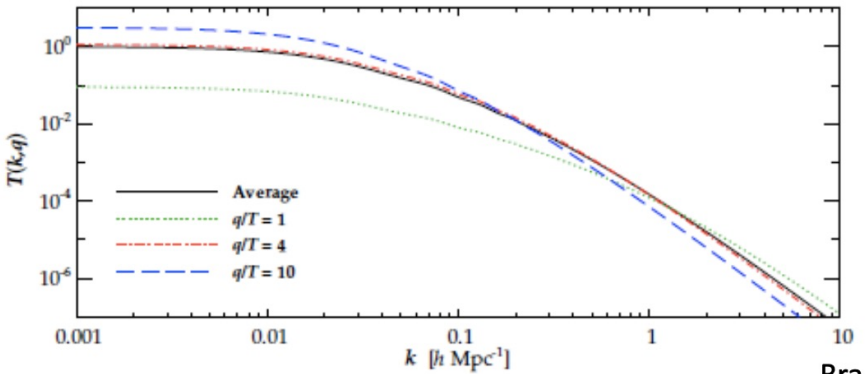
$$f = f_0 + \frac{\partial f_0}{\partial T}\delta T = f_0(1+\Psi) \qquad f_0(q) = \frac{1}{e^{q/T}+1}$$

After neutrino decoupling CBE

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = 0$$

$$\delta\rho_\nu(k) = 4\pi a^{-4}\int q^2 dq \epsilon f_0 \Psi_0 \qquad \epsilon = (q^2 + a^2 m^2)^{1/2}$$

$$\dot{\Psi}_0 = -\frac{qk}{3\epsilon}\Psi_1 - \dot{\phi}\frac{d\ln f_0}{d\ln q},$$
$$\dot{\Psi}_1 = \frac{qk}{\epsilon}\left(\Psi_0 - \frac{2}{5}\Psi_2\right) - \frac{\epsilon k}{q}\psi\frac{d\ln f_0}{d\ln q},$$
$$\dot{\Psi}_l = \frac{qk}{\epsilon}\left(\frac{l}{2l-1}\Psi_{l-1} - \frac{l+1}{2l+3}\Psi_{l+1}\right), \; l \geq 2$$

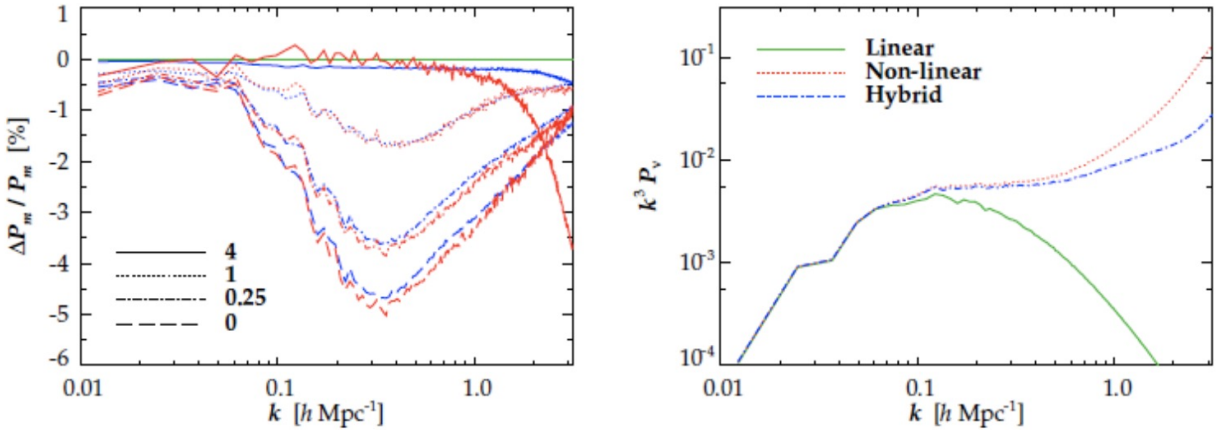


$$\Psi_0(k,q,z) = T(k,q,z)\Psi_0^I(k,q)$$

Brandbyge & Hannestad 09

Non-linear matter power

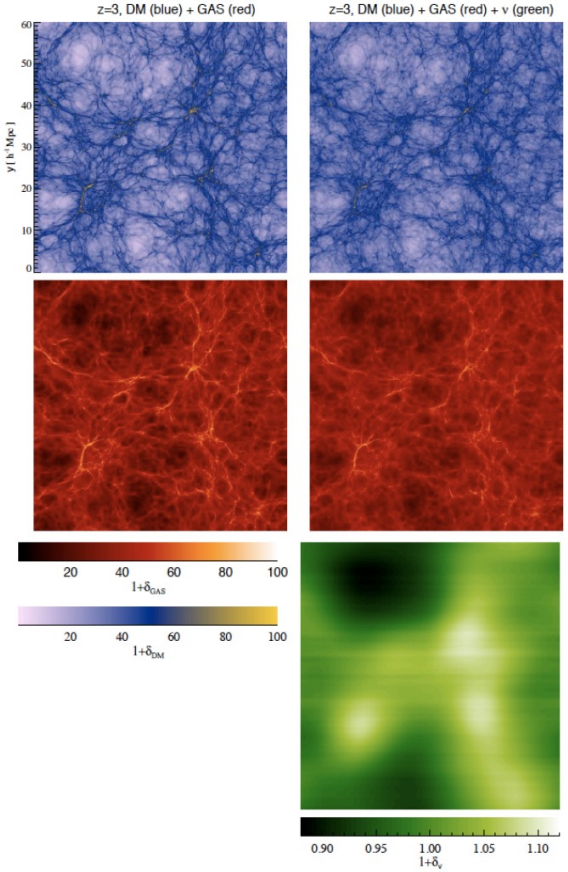
Matteo Viel



PARTICLES: accurate non-linear sampling but prone to shot-noise errors

GRID: fast and accurate but no phase mixing (i.e. non-linear regime suppression maybe it is less than it should be)

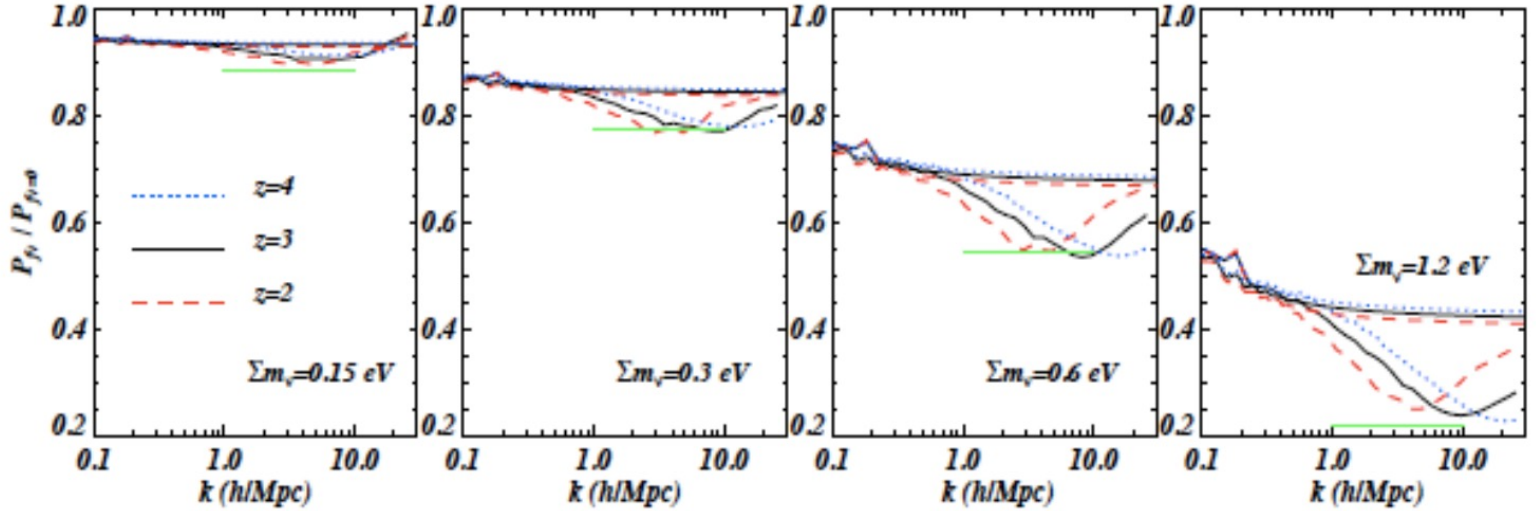
First hydro sim



TreeSPH code Gadget-III
follows DM, neutrinos, gas and star particles in a cosmological volume

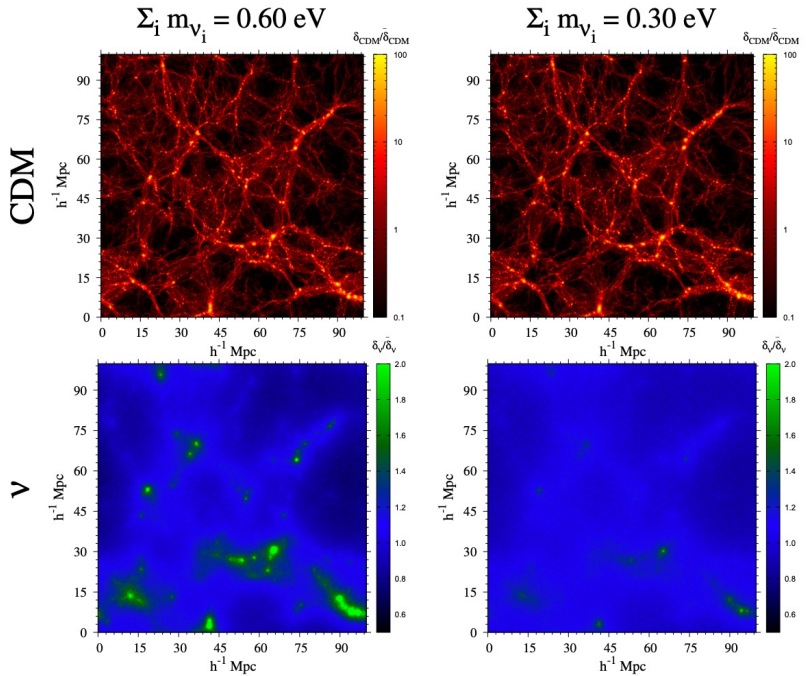
Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

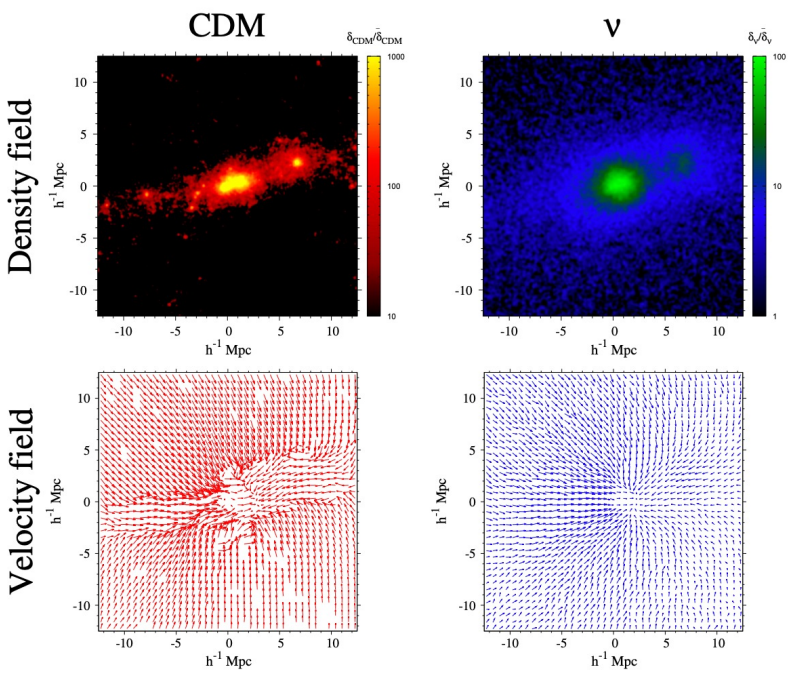
Total matter power suppression



Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

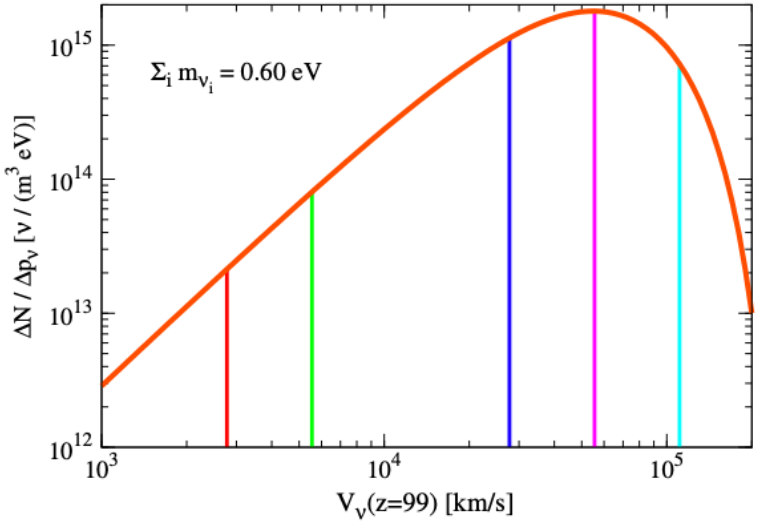
Neutrino fluid clustering

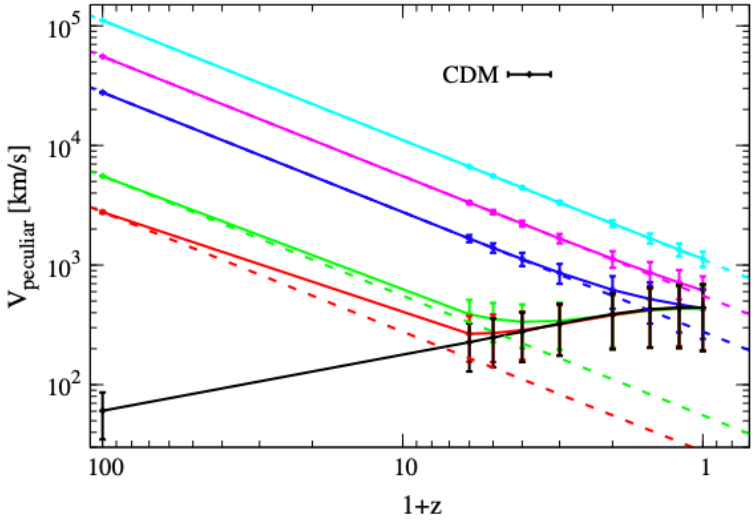




Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

Neutrino peculiar velocity evolution

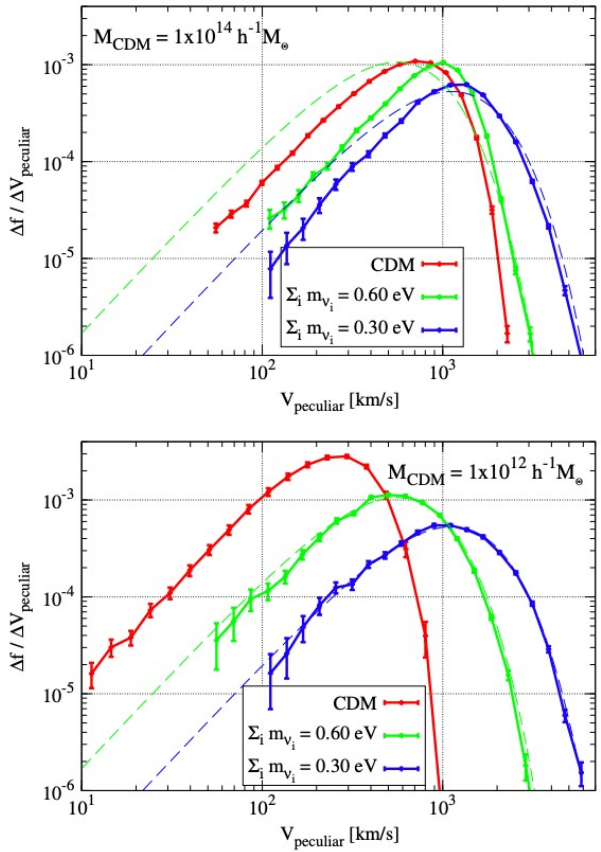




Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

Neutrino peculiar velocity evolution – II

Matteo Viel

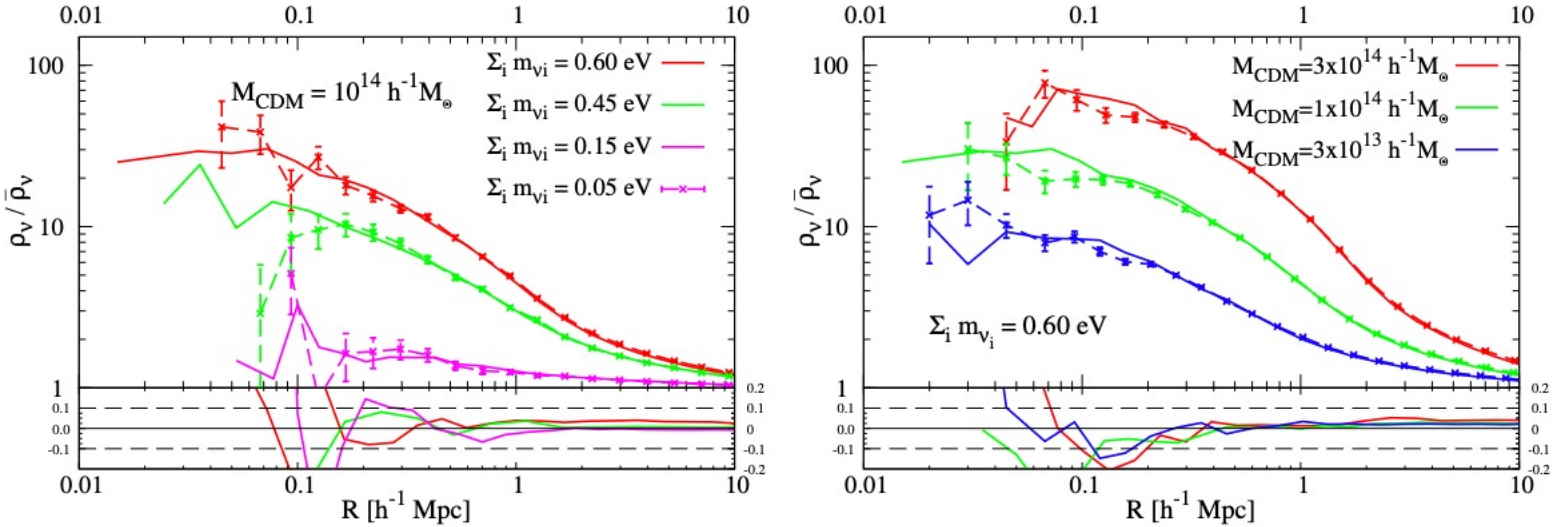


This is by selecting neutrino particles around haloes of a given mass

Neutrino slow down and cluster in a “selective” way

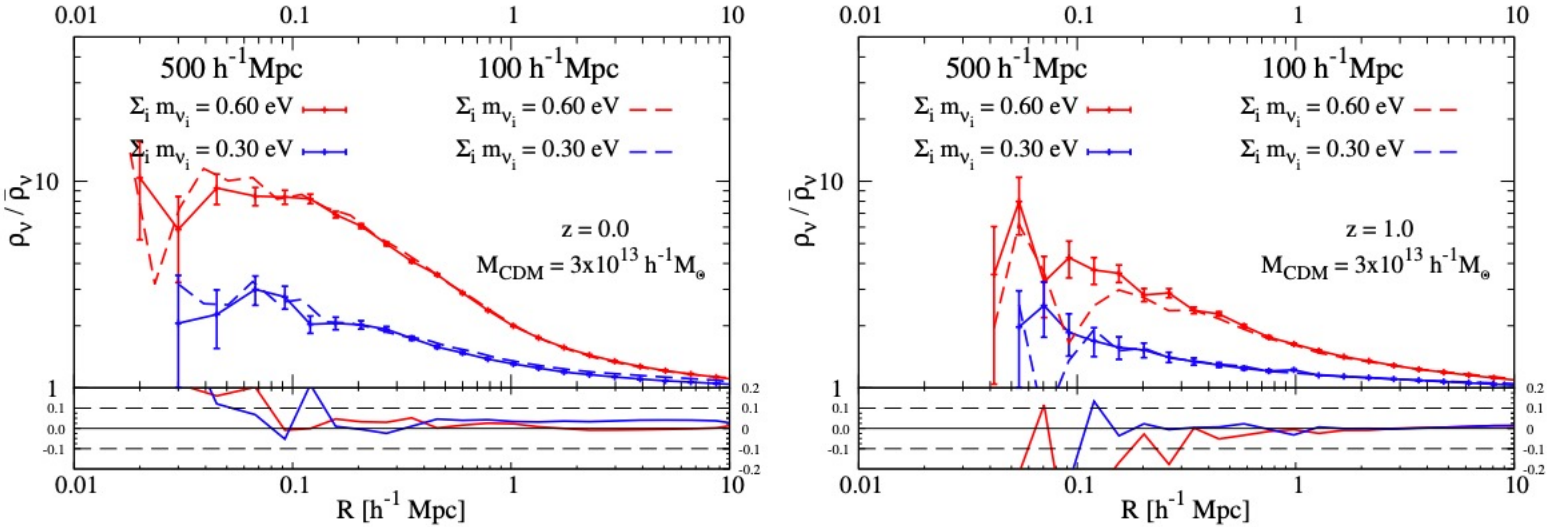
Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

The neutrino halo



Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

The neutrino halo



Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

Mass functions - I

Castorina, Sefusatti, Sheth, Villaescusa-Navarro, Viel 2013

FoF halos : b=0.2

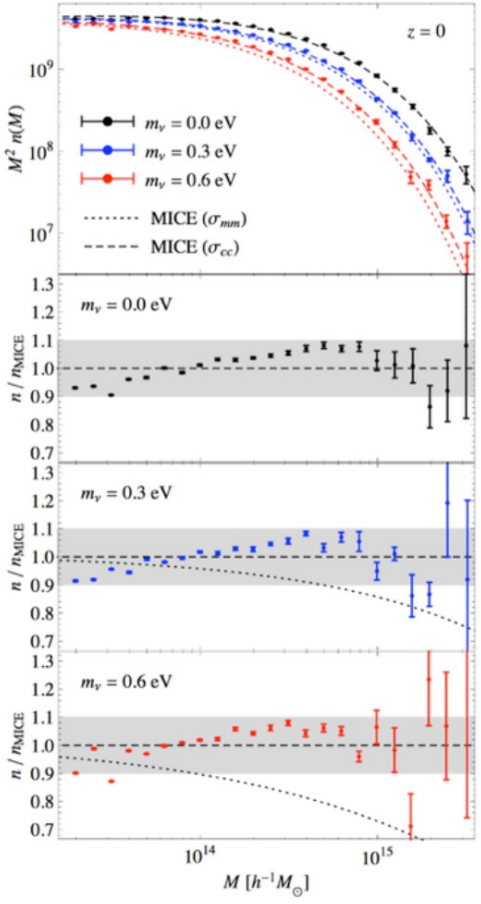
$$\frac{dn(M,z)}{dM} = \underbrace{\nu\, f(\nu)\frac{\rho_m}{M^2}\frac{d\ln\nu}{d\ln M}}$$

Matter prescription

$\rho_m \to \rho_{cdm} \quad P_m(k) \to P_m(k)$

Cold dark matter prescription

$\rho_m \to \rho_{cdm} \quad P_m(k) \to P_{cdm}(k)$



Note that

$$P_{cdm}(k,z) \geq P_m(k,z)$$

Mass functions - II

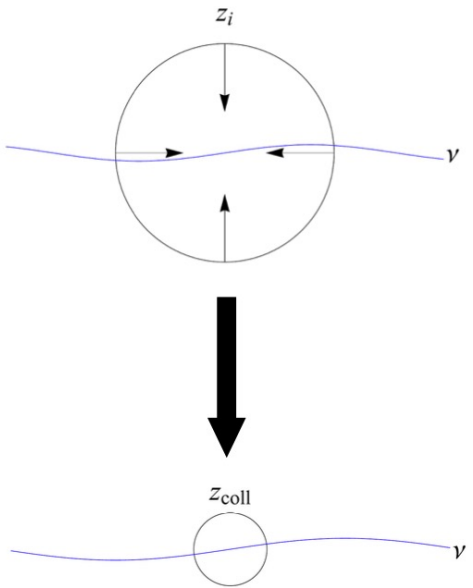
$$\delta_{cdm} > \delta_{crit} + a\,\delta_{\nu}$$

The physical picture:

the free streaming length is much larger than Lagrangian size of halos, neutrino perturbations do not play any role in the collapse.

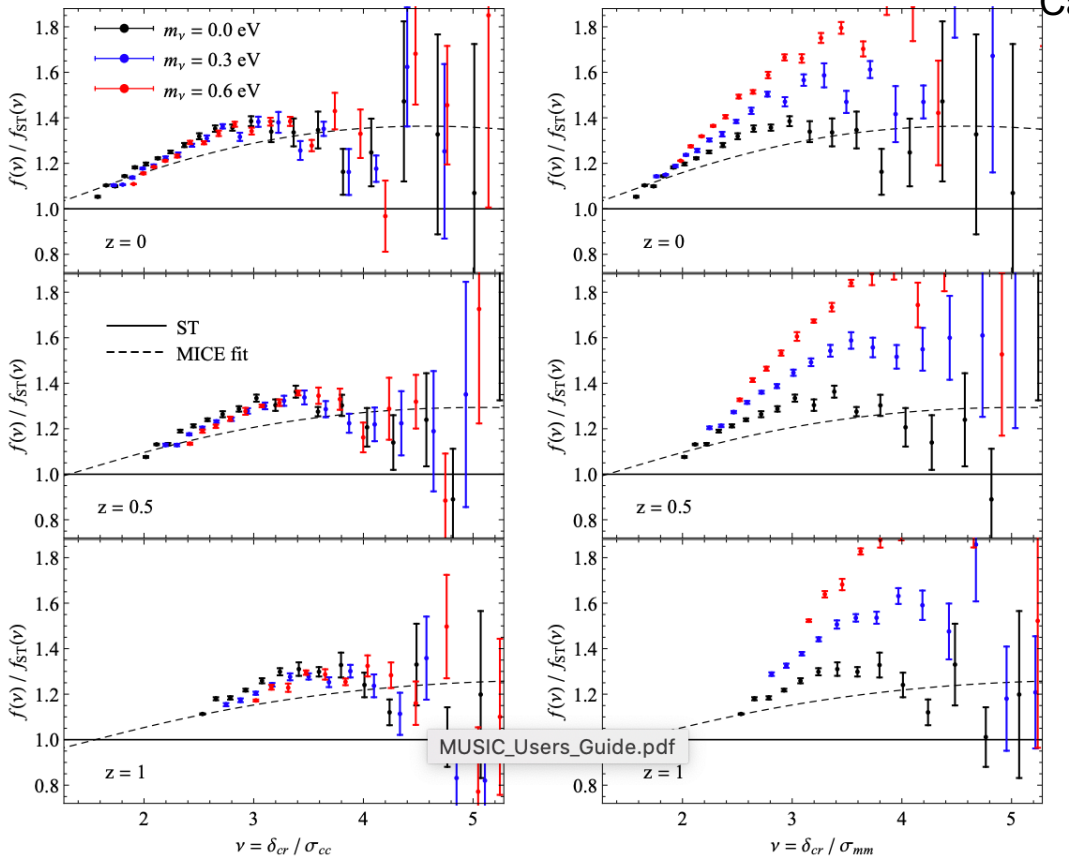
Ichiki&Takada(2012) studied the spherical collapse with massive neturinos, finding sub % effect on the collapse threshold.

They can be treaded as a background cosmology effect, like a Cosmological Constant, and we can and should use the CDM power spectrum.



Mass functions - III

Castorina+14



MUSIC_Users_Guide.pdf

Halo bias?

Halos and galaxies are biased tracers of the underlying mass distribution

$$\delta_h(x) = b\,\delta_m(x)$$

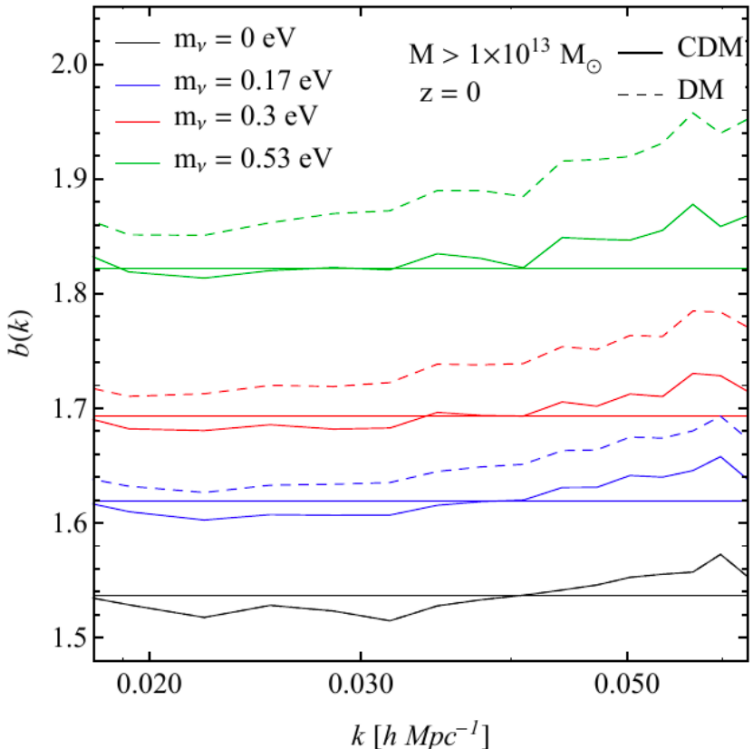
Linear bias is expected to be scale-independent on large scales.

$$b_c^{(hh)} \equiv \sqrt{\frac{P_{hh}(k)}{P_{cc}(k)}}$$

$$b_m^{(hh)} \equiv \sqrt{\frac{P_{hh}}{P_{mm}}} = b_c^{(hh)} \sqrt{\frac{P_{cc}}{P_{mm}}}$$

Potential systematic error in galaxy clustering measurements.

Castorina+14



Main conclusions (so far)

- N-body simulations recover linear growth at large scale
- Non-linear regime presents an overall suppression of power which is about 25% larger than in linear theory
- There is a distinctive spoon-shape feature at small scales, however, this is not due to neutrinos, but it is due to the different growth of small vs. large scales and their relative coupling → it can be used to constrain neutrinos once amplitude at larger scales is fixed
- Neutrino halo is characterized and it is cored for haloes $> 10^{13.5}$ Msun/h
- FD distribution of neutrinos is perturbed inside virialized haloes
- Momentum distribution is also perturbed over large cosmological volumes (the more massive they are the closer they are to CDM)

Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

Are numerical effects under control?

Euclid*: Modelling massive neutrinos in cosmology – a code comparison

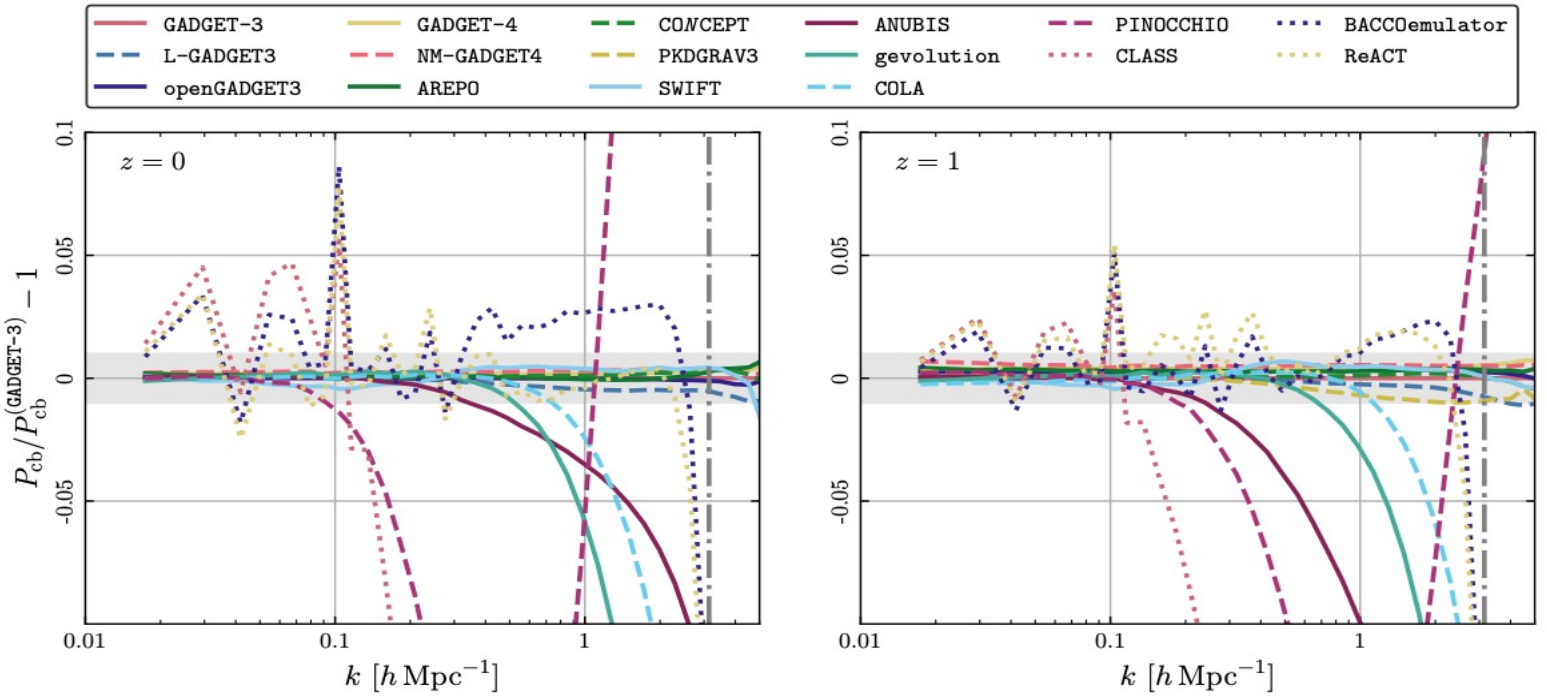


J. Adamek,[1] R. E. Angulo,[2,3] C. Arnold,[4] M. Baldi,[5,6,7]

Code	type	neutrino method(s)
GADGET-3	N-body (Tree-PM)	particle
L-GADGET3	N-body (Tree-PM)	mesh
openGADGET3	N-body (Tree-PM)	particle
GADGET-4	N-body (Tree-PM)	particle
NM-GADGET4	N-body (Tree-PM)	Newtonian motion gauge
AREPO	N-body (moving mesh)	particle
CO*N*CEPT	N-body ($\mathrm{P^3M}$)	mesh
PKDGRAV3	N-body (Tree + FMM)	mesh
SWIFT	N-body (PM + FMM)	particle / δf
ANUBIS	N-body (PM + AMR)	particle
gevolution	N-body (uniform PM)	particle / mesh
COLA	N-body surrogate	mesh
PINOCCHIO	N-body surrogate	linear growth factor
ReACT	$P(k)$ prediction	halo-model reaction
BACCOemulator	$P(k)$ prediction	emulation
EuclidEmulator2	$P(k)$ prediction	emulation
Cosmic Emu	$P(k)$ prediction	emulation

Are numerical effects under control?

Matteo Viel



Halo model

$$\rho(\mathbf{x}) = \sum_i \mathcal{N}_i M_i u(\mathbf{x}-\mathbf{x}_i|M_i)$$

$$\begin{aligned}\langle\rho(\mathbf{x})\rangle &= \int \mathrm{d}M\, M n(M) \sum_i \Delta V_i\, u(\mathbf{x}-\mathbf{x}_i|M) \\ &= \int \mathrm{d}M\, M n(M) \int \mathrm{d}^3\mathbf{x}' u(\mathbf{x}-\mathbf{x}'|M) \\ &= \int \mathrm{d}M\, M n(M) = \overline{\rho},\end{aligned}$$

$$\langle\rho(\mathbf{x}_1)\rho(\mathbf{x}_2)\rangle = \sum_{i,j}\langle\mathcal{N}_i M_i \mathcal{N}_j M_j u(\mathbf{x}_1-\mathbf{x}_i|M_i) u(\mathbf{x}_2-\mathbf{x}_j|M_j)\rangle.$$

$$\xi(r) \equiv \langle\delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle = \xi^{1\mathrm{h}}(r) + \xi^{2\mathrm{h}}(r) \qquad (r \equiv |\mathbf{x}_1-\mathbf{x}_2|),$$

where

$$\xi^{1\mathrm{h}}(r) = \frac{1}{\overline{\rho}^2}\int \mathrm{d}M\, M^2 n(M) \int \mathrm{d}^3\mathbf{y}\, u(\mathbf{y}-\mathbf{x}_1|M) u(\mathbf{y}-\mathbf{x}_2|M),$$

$$\begin{aligned}\xi^{2\mathrm{h}}(r) = \frac{1}{\overline{\rho}^2}\int \mathrm{d}M_1\, M_1 n(M_1) \int \mathrm{d}M_2\, M_2 n(M_2) \int \mathrm{d}^3\mathbf{x} \int \mathrm{d}^3\mathbf{x}' \\ \times\, u(\mathbf{x}_1-\mathbf{x}|M_1) u(\mathbf{x}_2-\mathbf{x}'|M_2) \xi_{\mathrm{hh}}(\mathbf{x}-\mathbf{x}'|M_1, M_2).\end{aligned}$$

Mo, Van den Bosch & White
”Galaxy formation and evolution”
book

Halo model

$$\rho(\mathbf{k}) = \sum_i \mathcal{N}_i M_i \tilde{u}(\mathbf{k}|M_i) e^{-i\mathbf{k}\cdot\mathbf{x}_i}.$$

Here $\tilde{u}(\mathbf{k}|M_i)$ is the Fourier transform of the density profile, which for a spherically profile truncated at the virial radius $r_{\rm h}$ is given by

$$\tilde{u}(k|M) = \frac{4\pi}{M}\int_0^{r_{\rm h}} \rho(r|M)\,\frac{\sin kr}{kr}\,r^2\,{\rm d}r.$$

$$P(k) \equiv V_{\rm u}\langle|\delta_{\mathbf{k}}|^2\rangle = P^{\rm 1h}(k) + P^{\rm 2h}(k),$$

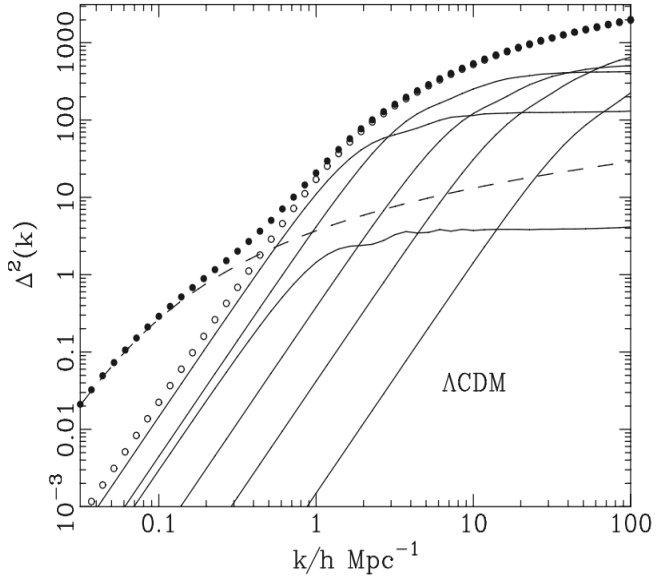
where

$$P^{\rm 1h}(k) = \frac{1}{\overline{\rho}^2}\int {\rm d}M\, M^2 n(M)\,|\tilde{u}(k|M)|^2\,,$$

$$P^{\rm 2h}(k) = \frac{1}{\overline{\rho}^2}\int {\rm d}M_1\, M_1 n(M_1)\tilde{u}(k|M_1)\int {\rm d}M_2\, M_2 n(M_2)\tilde{u}(k|M_2)P_{\rm hh}(k|M_1,M$$

$$P_{\rm hh}(k|M_1,M_2) = b_{\rm h}(M_1)\,b_{\rm h}(M_2)\,P_{\rm lin}(k),$$

$$P^{\rm 2h}(k) = P_{\rm lin}(k)\left[\frac{1}{\overline{\rho}}\int {\rm d}M\, M\, n(M)\, b_{\rm h}(M)\,\tilde{u}(k|M)\right]^2$$



A neutrino halo model

$$P(k) = \left(\frac{\bar{\rho}_c}{\bar{\rho}}\right)^2 P_c(k) + 2\,\frac{\bar{\rho}_c\bar{\rho}_\nu}{\bar{\rho}^2}\,P_{c\nu}(k) + \left(\frac{\bar{\rho}_\nu}{\bar{\rho}}\right)^2 P_\nu(k)$$

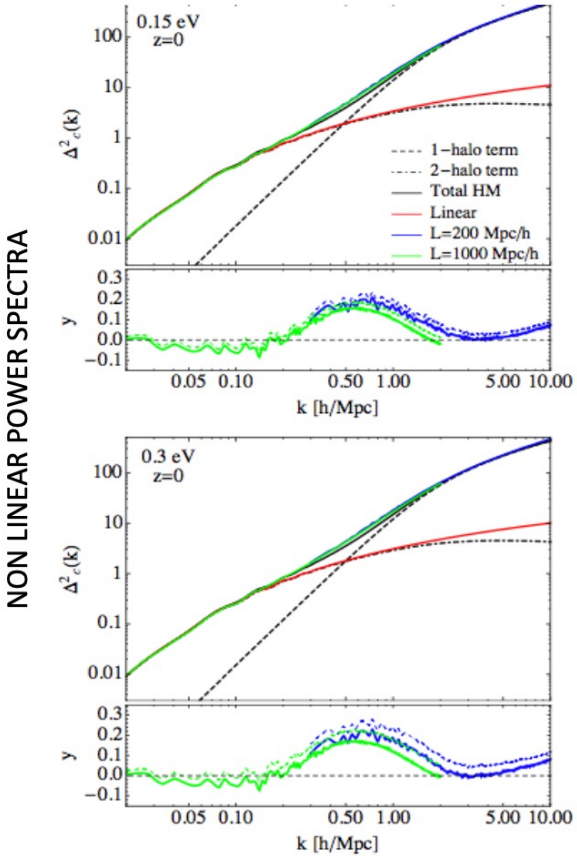
- Assumption: all matter within haloes 1h and 2h terms

- Simple modelling of non-linear power spectra (including cross-spectra)

- When used to predict ratios w.r.t. massless case it is as good as hydro/N-body to 2% level

- When used to compute actual power it suffers from limitation and it is good at the 20% level

NON LINEAR POWER SPECTRA



Massara, Villaescusa, MV (2014) – Castorina+ (2014) for bias and mass functions

Perturbation theory

PT results (I)

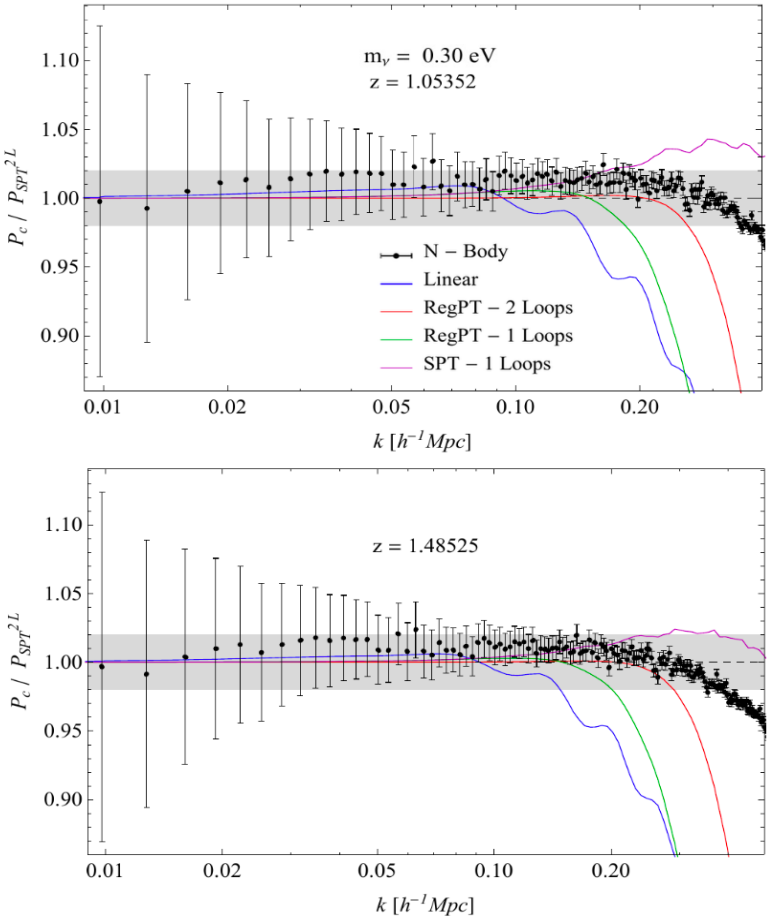
Non linear just in the CDM component.

Neglect scale dependent growth factor, see Blas+14

PT works at mildly non linear scales as in LCDM cosmology

At z=1.5 PT is accurate at a few % level up to kmax = 0.4 h/Mpc

What about Pmm?



Summary of Lecture 2

Neutrino non-linear clustering is selective: depends on halo mass, redshift, scales

This is good: quantitative predictions in a variety of environments,
mass functions, Bias of haloes/galaxis, neutrino haloes

All the effects are somewhat at the percent or sub-percent level

Precision post-Planck cosmology will address this

Plan

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Some thoughts

- Neutrinos do not cluster much
- Non-linearities produce features in the massive neutrino Matter power spectrum but… non-linear scales are difficult to model (expensive N-body) and on top of that affected by baryonic processes (lecture #4)
- Maybe we should look at them in the high-z Universe, after CMB but before structures become too non-linear

IDEAL place intergalactic medium

Bibliography

References:

1) MODEL BUILDING:

Bi & Davidsen 1997,
"Evolution of Structure in the Intergalactic Medium and the Nature of the Lyα Forest", ApJ, 479, 523

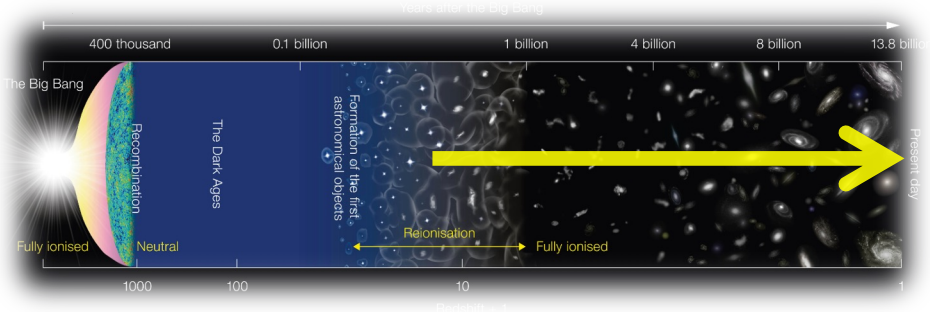
2) MORE ON OBSERVATIONS:

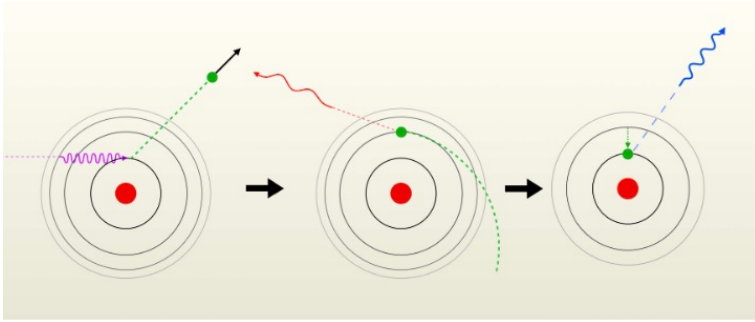
Rauch, 1998, "The Lyman-alpha forest in the spectra of QSOs", ARA&A, 32, 267

3) RECENT REVIEWS (include sims and recent data sets):

Meiksin, 2009, "The Physics of the IGM", Progress Reports, 81, 1405 McQuinn, 2016, "The Evolution of the Intergalactic Medium", ARA&A, 54,313

Intergalactic Medium





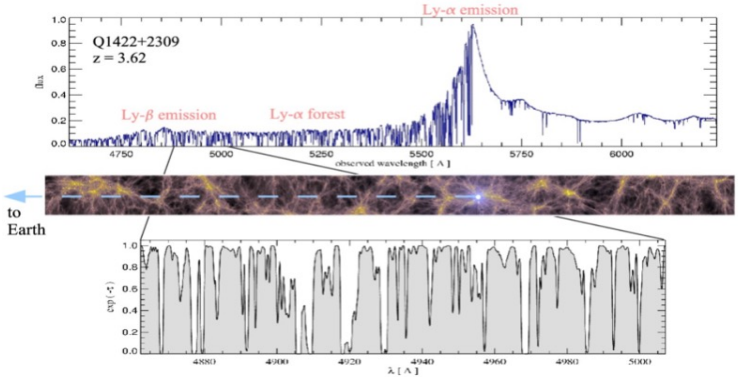
Post-reionization Universe

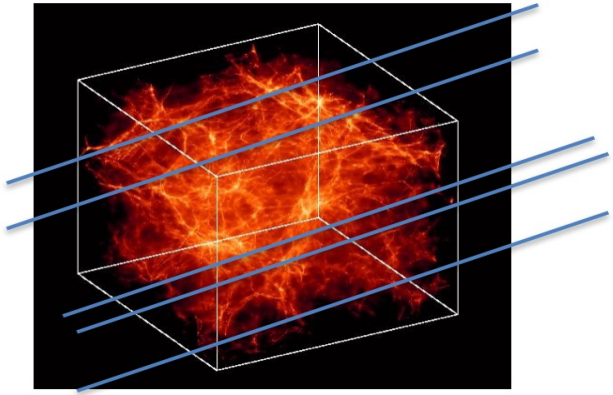
- Complementary to Cosmic Microwave Background (CMB) and local probes
- More linear Universe (simpler physics?)
- High-z galaxies are cold gas (HI) dominated
- Large **uncharted** volume: JWST, LSST, Euclid, DESI, Intensity Mapping (IM) experiments

Intergalactic Medium

Matteo Viel

The Lyman-alpha forest



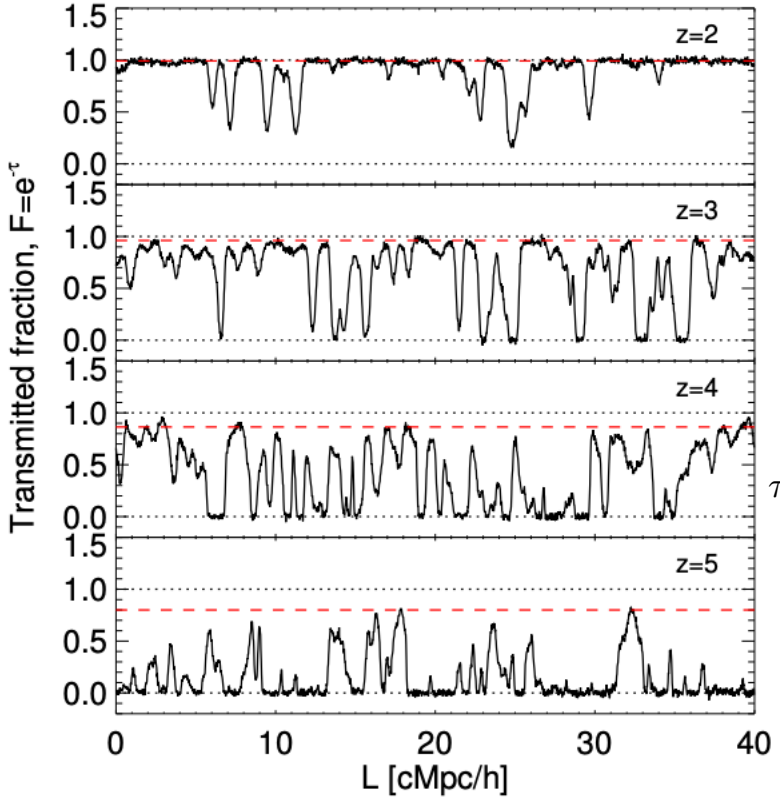


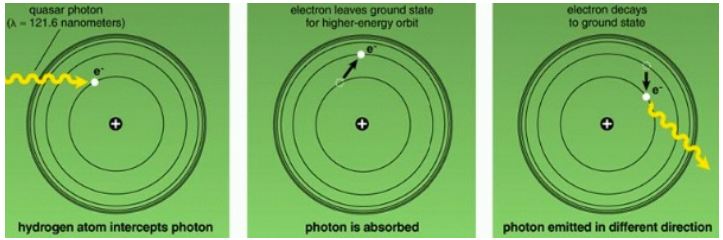
- **Intergalactic medium:** filaments at low density (outside galaxies) - distances spanned 0.1-100 Mpc/h
- Lyman-alpha forest its the main manifestation of the IGM
- High redshift observable, 1D projected power (but also 3D)

Physics of the Lyα forest

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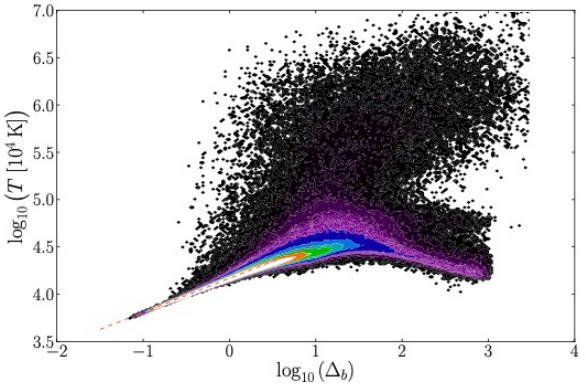
High-z (2<z<6) cosmic web





Shape of the lines determined by HI abundance, IGM thermal state, etc.

$$\tau_{\rm HI} \propto \Delta_b \, x_{\rm HI} \propto \Delta_b^2 \, T^{-0.7} / \Gamma_{\rm HI}$$



Physics of the Lyman-a forest

(Bi 1993, Bi & Davidsen 1997, Hui & Gnedin 1998, Matarrese & Mohayaee 2002)

$$k_J^{-1}(z) \equiv H_0^{-1}\left[\frac{2\gamma k_B T_m(z)}{3\mu m_p \Omega_{0m}(1+z)}\right]^{1/2}$$ Jeans length

$$\delta_0^{\rm IGM}(\mathbf{k},z) = \frac{\delta_0^{\rm DM}(\mathbf{k},z)}{1+k^2/k_J^2(z)} \equiv W_{\rm IGM}(k,z) D_+(z)\delta_0^{\rm DM}(\mathbf{k})$$ Filtering of linear DM density field

$$\mathbf{v}^{\rm IGM}(\mathbf{k},z) = E_+(z)\frac{i\mathbf{k}}{k^2} W_{\rm IGM}(k,z)\delta_0^{\rm DM}(\mathbf{k})$$ Peculiar velocity

$$n_{\rm IGM}(\mathbf{x},z) = \overline{n}_{\rm IGM}(z)\exp\left[\delta_0^{\rm IGM}(\mathbf{x},z) - \frac{\langle(\delta_0^{\rm IGM})^2\rangle D_+^2(z)}{2}\right]$$ Non linear density field

$$T(\mathbf{x},z) = T_0(z)\,(1+\delta^{\rm IGM}(\mathbf{x},z))^{\gamma(z)-1}$$ 'Equation-of-state'

$$\alpha(z,T(z))\, n_p n_e = J(z) n_{\rm HI}\ ,$$ Neutral hydrogen ionization equilibrium equation

$$\tau(u) = \frac{\sigma_{0,\alpha}\, c}{H(z)} \int_{-\infty}^{\infty} dy\, n_{\rm HI}(y)\, \mathcal{V}\left[u - y - v_\parallel^{\rm IGM}(y),\, b(y)\right]$$ Optical depth

Density Velocity Temperature

Linear fields: density, velocity

Non linear fields

+

Temperature

Spectra: Flux=exp(-τ)

MV, Matarrese S., Mo HJ., Haehnelt M., Theuns T., 2002a, MNRAS, 329, 848

More recent milestones

DATA: early 90s: advent of high res spectroscopy (UVES, Keck)

[1998-2002] Croft, Weinberg+: first quantitative use of the Lyman-alpha forest for cosmology.

[1998-2004] better understanding of physics of the IGM (Hui, Gnedin, Meiksin, White).

[2004] Viel+: usage of UVES to complement Croft's work with better sims to cover the parameter space.

[2005-06] SDSS-II results (McDonald, Seljak…): excellent synergy with CMB abd other probes demonstrated (constraints on inflation and neutrinos).

[2007-now] systematic use of QSO spectra for DM nature at small scales (Viel+).

[2013] BAO detected in the Lyman-alpha forest 3D correlation by BOSS (SDSS-III) from low resolution.

Modelling the observables

Matteo Viel

Modelling 3D flux power

Arino-i-Prats, Miralda-Escude', MV, Cen (2015) - McDonald (2003)

$$\delta_F = \frac{F}{\bar{F}(z)} - 1 \qquad \eta = -\frac{1}{aH}\frac{\partial v_p}{\partial x_p}$$

$$\beta = \frac{b_{F\eta}\, f(\Omega)}{b_{F\delta}} \qquad b_{F\delta} = \frac{\partial \delta_F}{\partial \delta} \qquad b_{F\eta} = \frac{\partial \delta_F}{\partial \eta}$$

$$\delta_F = b_{F\delta}\delta + b_{F\eta}\eta$$

3D flux power

$$P_F(k,\mu) = b_{F\delta}^2\,(1+\beta\mu^2)^2\; P_L(k)\, D(k,\mu)$$

$$D\,(k,\mu) = \exp\left\{ \left[q_1\Delta^2(k) + q_2\Delta^4(k)\right] \left[1 - \left(\frac{k}{k_v}\right)^{a_v} \mu^{b_v}\right] - \left(\frac{k}{k_p}\right)^2 \right\}$$

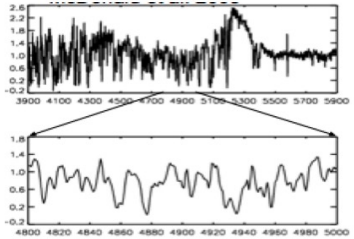
non-linear matter power | thermal broadening | pressure smoothing

1D flux power

$$P_{1D}(k_\parallel, z) = \frac{1}{2\pi}\int_{k_\parallel} P_F(k, k_\parallel, z)\, k\, dk$$

Data

BOSS/SDSS-III



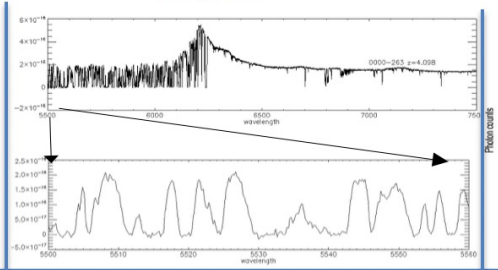
Low resolution BOSS and SDSS-III spectra S/N~2-3 - 160,000 spectra

Used to detect BAOs at z=2.3 and correlations in the transverse direction

Used to place stringent constraints on neutrino masses <0.12 eV

Busca+13, Slosar+14, Font-Ribera+14
Palanque-Delabrouille+15
Seljak+06, Baur+16, Yeche+17 etc.

XQ-100



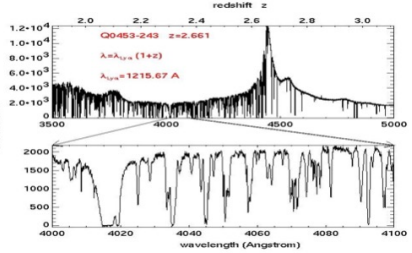
Medium resolution X-Shooter VLT spectra S/N ~ 30

100 spectra at z>3.5

Used to place stringent constraints on Warm Dark Matter in combination with high res. spectra

Irsic, MV+ 17a,17b
Lopez+16, Irsic+16

HIRES/MIKE



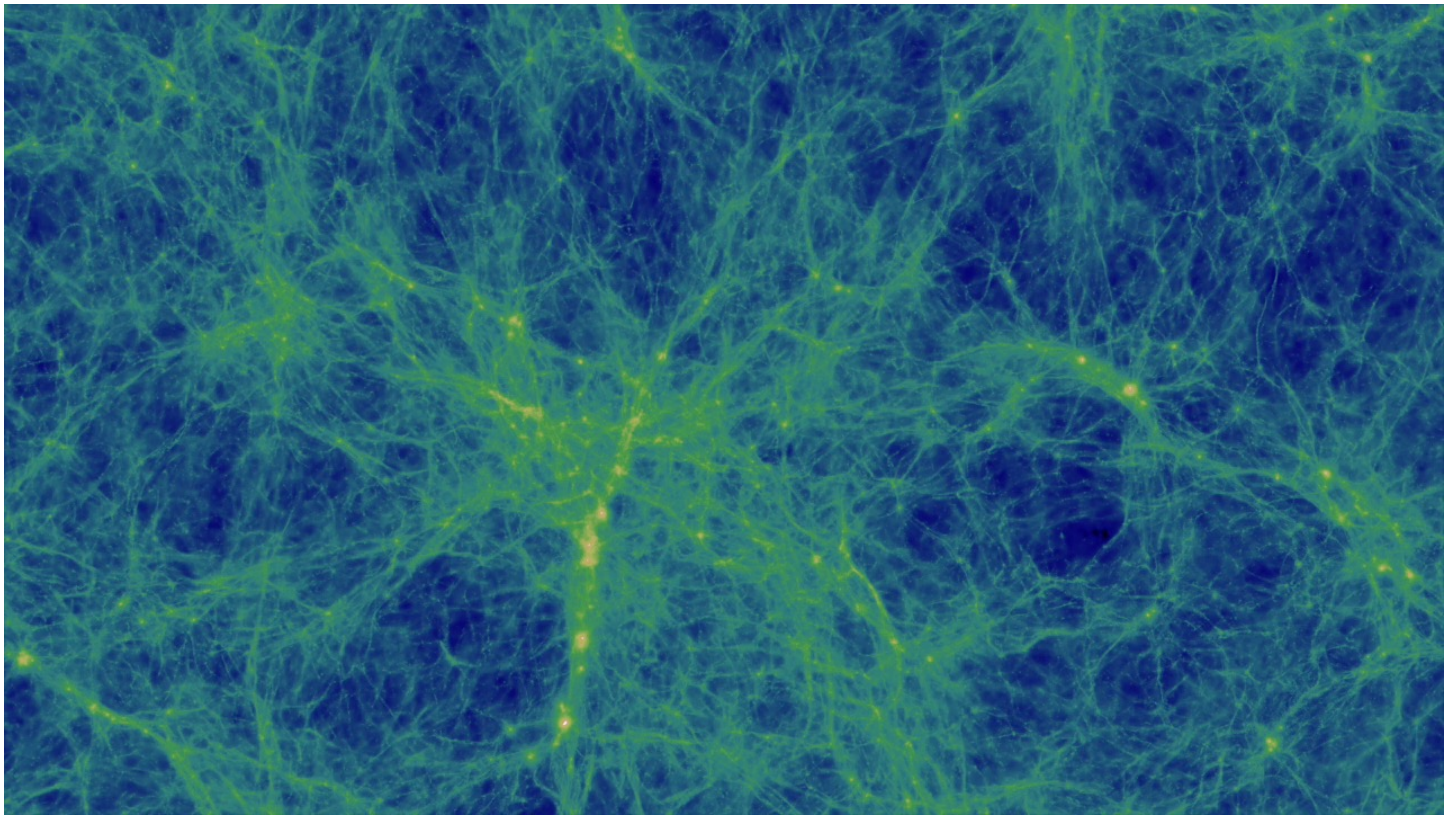
High resolution VLT or Keck spectra S/N ~100 - ~hundreds of spectra

Used for WDM, astrophysics of the IGM and galaxy formation, variation of fundamental constants

*MV+05,08,13, **Becker**+11*
Yeche+17, Garzilli+18 ,
Bosman+18

Movie nr. 2

Matteo Viel

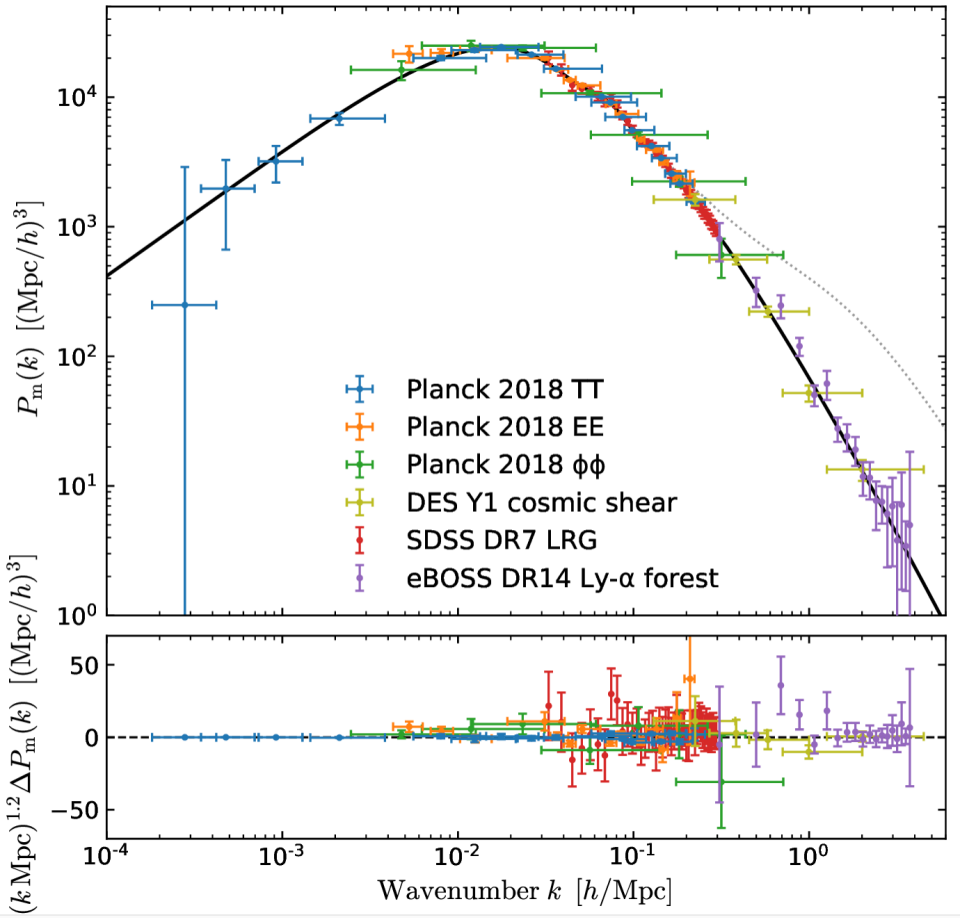


Bolton+17, Sherwood simulation suite (PRACE call: 15 CPU Mhrs)
Puchwen+19,+22, Sherwood relics (PRACE+Dirac call: 60 CPU Mhrs

Long lever arm of the linear power spectrum

Matteo Viel

Chabanier+19

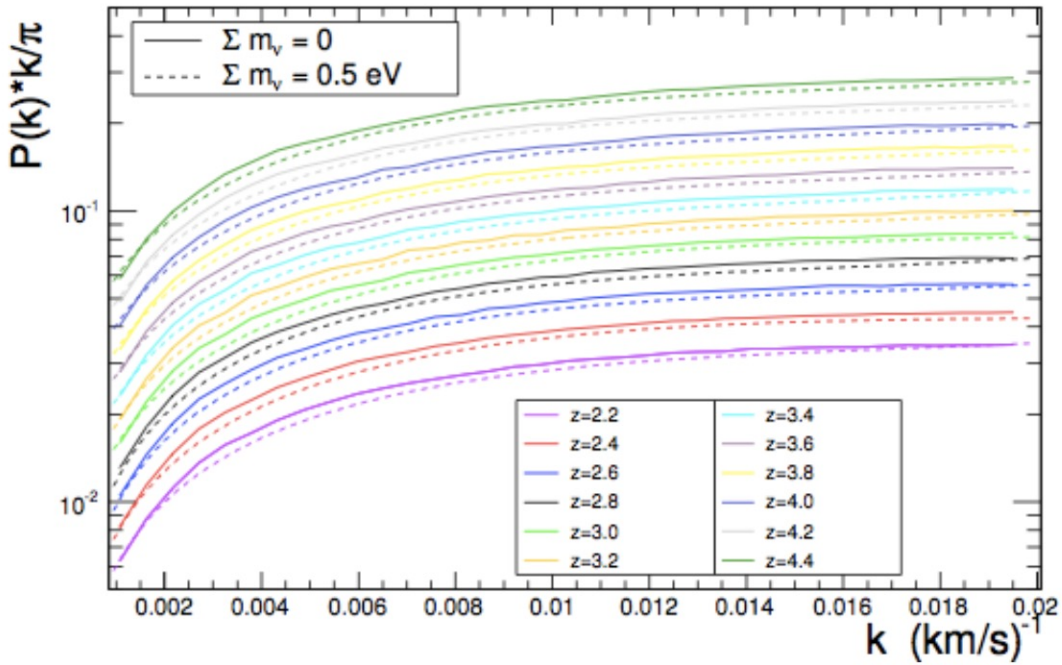


Two reasons for why Lyα is so constraining:

1) 1D is projected power.
2) We are at high-z possibly closer to linear regime.

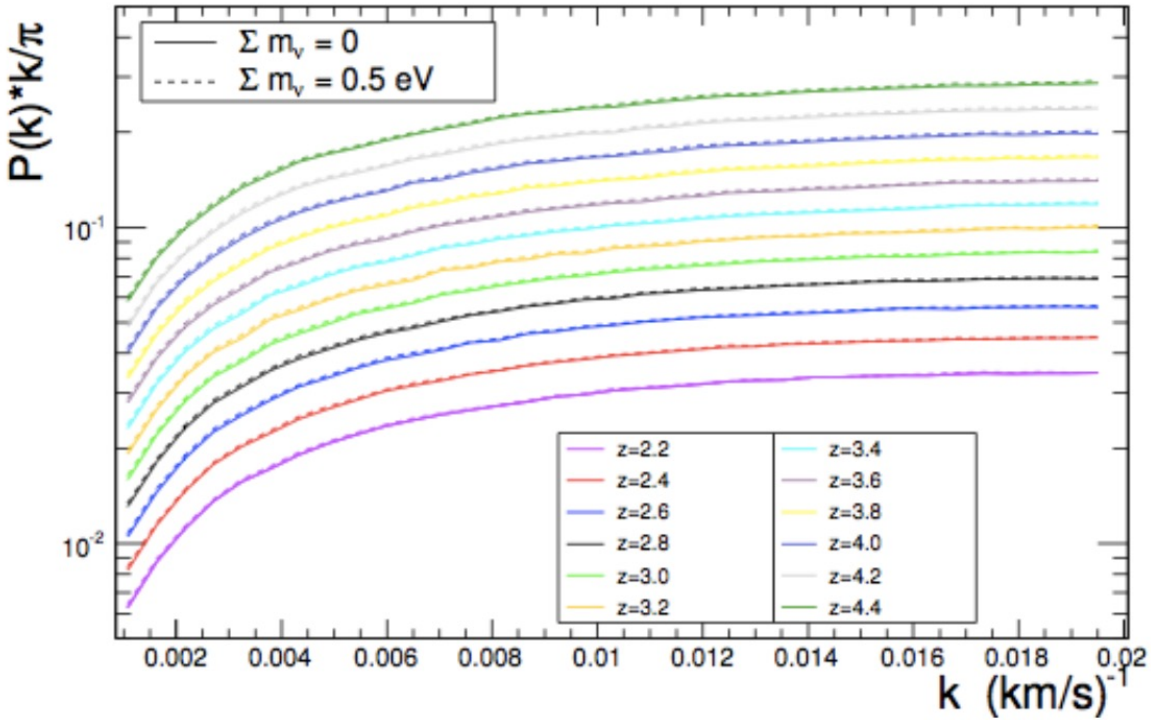
Neutrino impact - I

Matteo Viel



Neutrino impact - II

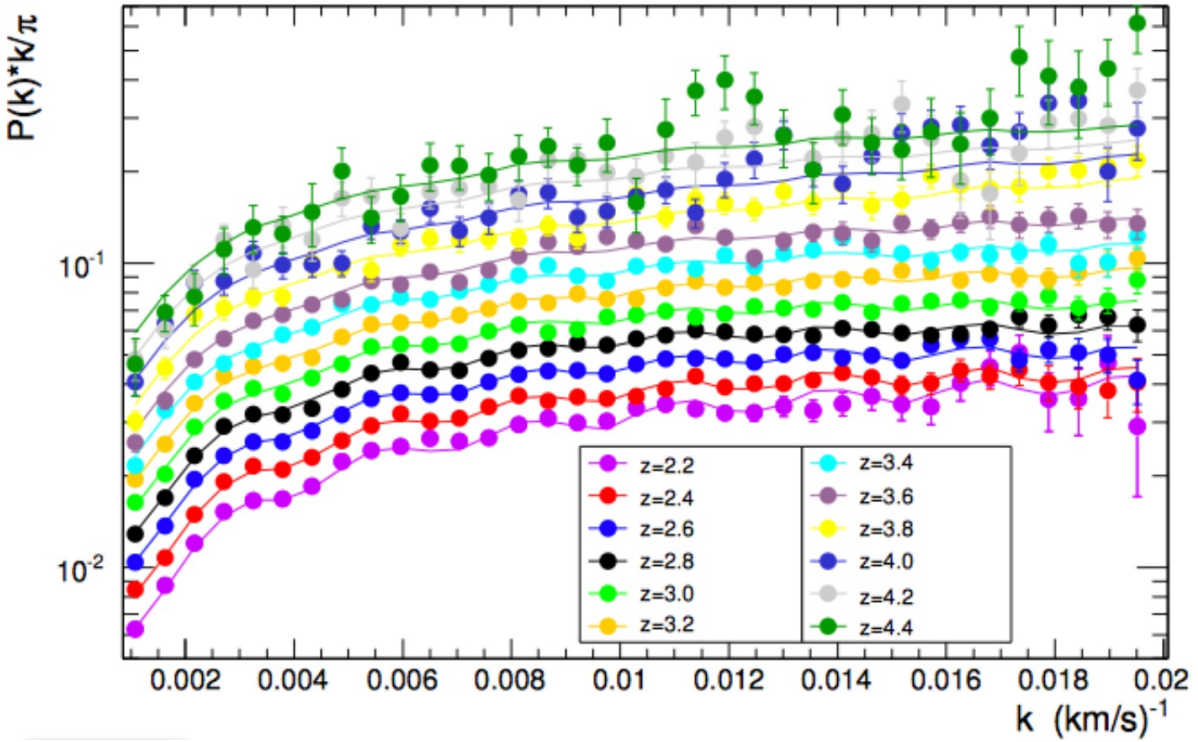
Matteo Viel



Neutrino impact - III

1D Flux power spectrum evolution

Nathalie Palanque-Delabrouille,[a,b] Christophe Yèche,[a] Julien Lesgourgues,[c,d,e] Graziano Rossi,[a,f] Arnaud Borde,[a] Matteo Viel,[g,h] Eric Aubourg,[i] David Kirkby,[j] Jean-Marc LeGoff,[a] James Rich,[a] Natalie Roe,[b] Nicholas P. Ross,[k] Donald P. Schneider,[l,m] David Weinberg[n]

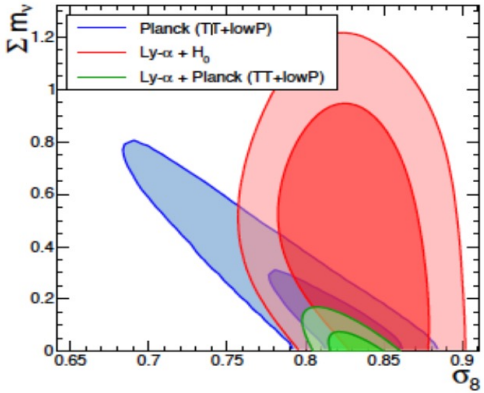


Neutrino impact - IV

UPDATE using Planck 15

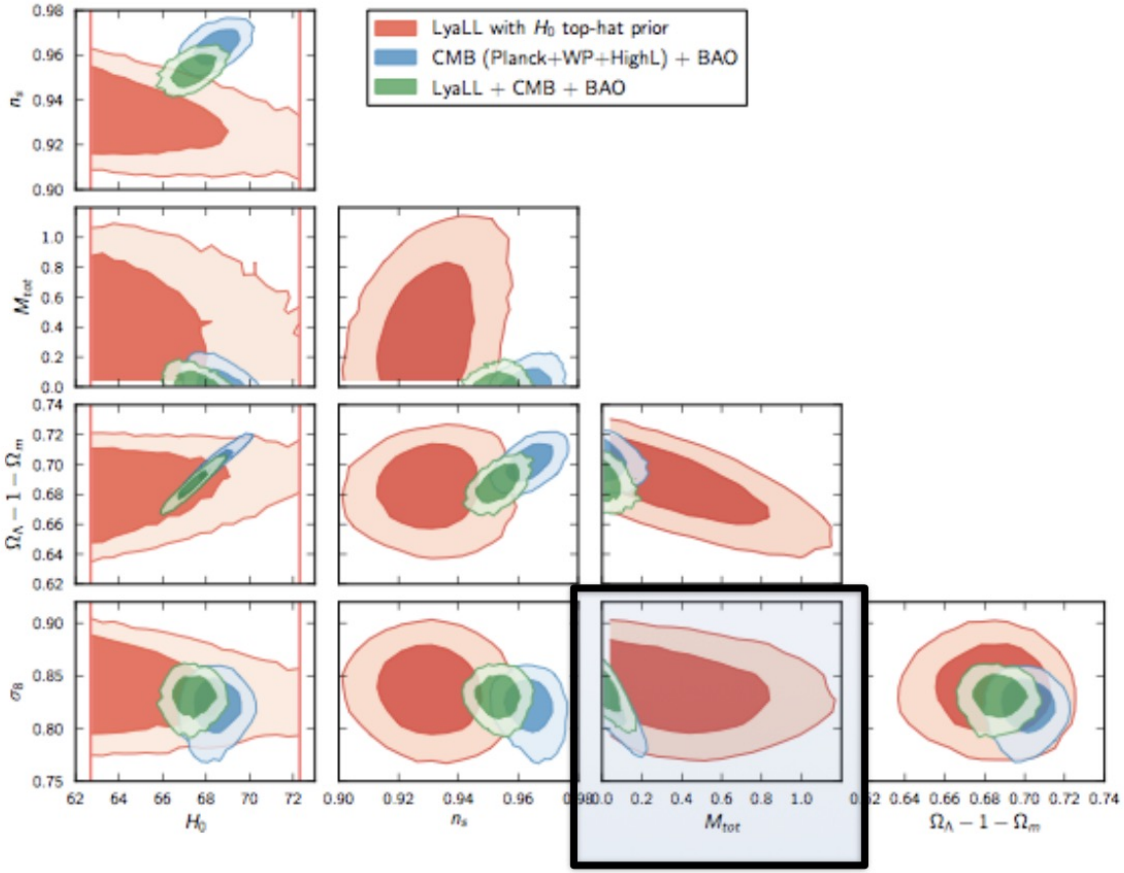
Palanque-Delabrouille+ 2015

Parameter	(1) Lyα + H_0^{Gaussian} ($H_0 = 67.3 \pm 1.0$)	(2) Lyα + Planck TT+lowP	(3) Lyα + Planck TT+lowP + BAO	(4) Lyα + Planck TT+TE+EE+lowP + BAO
σ_8	0.831 ± 0.031	0.833 ± 0.011	0.845 ± 0.010	0.842 ± 0.014
n_s	0.938 ± 0.010	0.960 ± 0.005	0.959 ± 0.004	0.960 ± 0.004
Ω_m	0.293 ± 0.014	0.302 ± 0.014	0.311 ± 0.014	0.311 ± 0.007
H_0 (km s^{-1} Mpc^{-1})	67.3 ± 1.0	68.1 ± 0.9	67.7 ± 1.1	67.7 ± 0.6
$\sum m_\nu$ (eV)	< 1.1 (95% CL)	< 0.12 (95% CL)	< 0.13 (95% CL)	< 0.12 (95% CL)
Reduced χ^2	0.99	1.04	1.05	1.05



Neutrino impact - IV

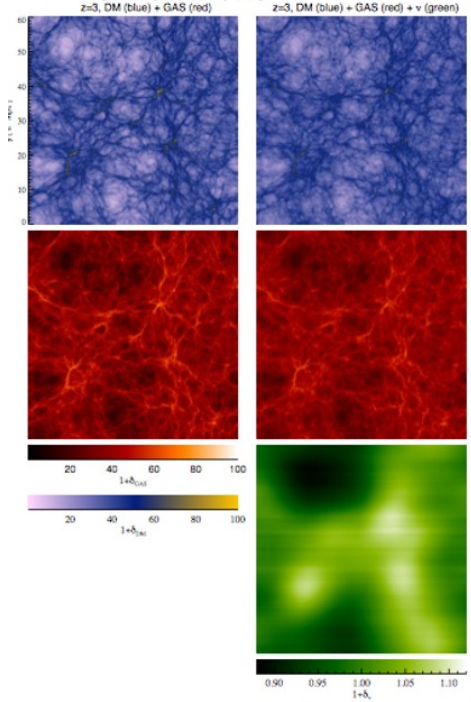
Matteo Viel

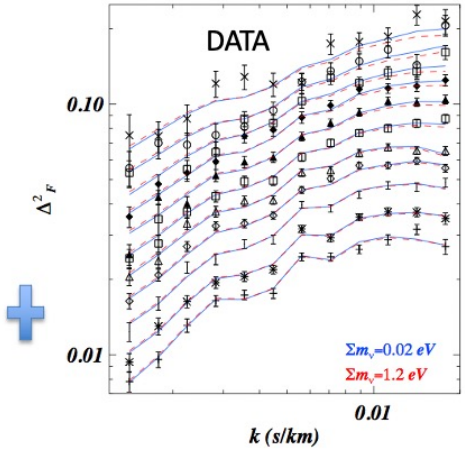


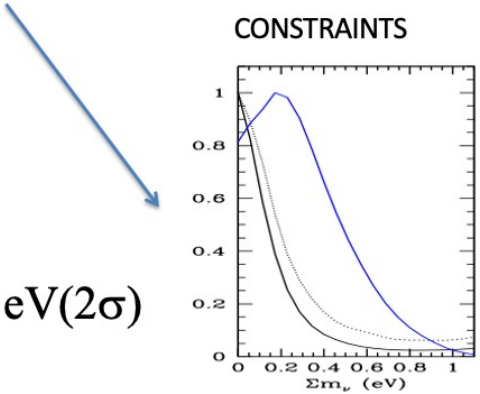
IGM and neutrinos

NEUTRINOS IN THE IGM

SIMULATIONS





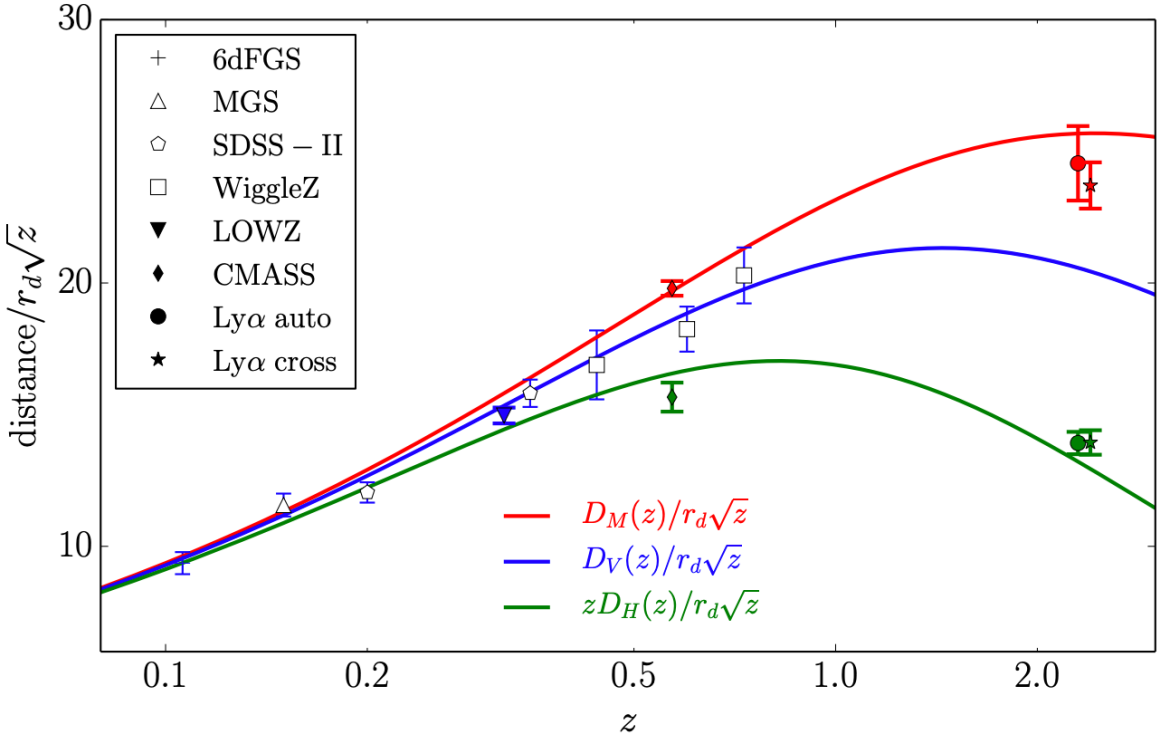


$\Sigma m_\nu < 0.9$ eV(2σ)

Viel, Haehnelt, Springel 2010

Neutrinos and BAOs

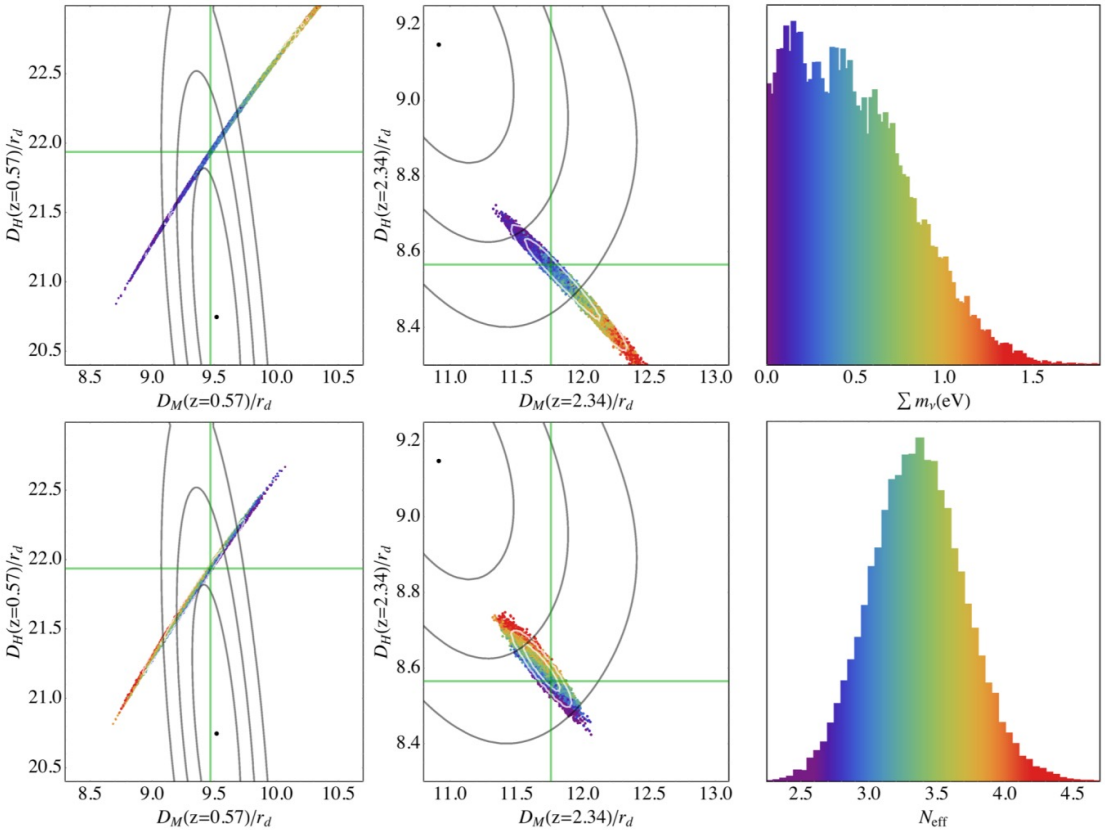
Aubourg+14



Neutrinos and BAOs

Matteo Viel

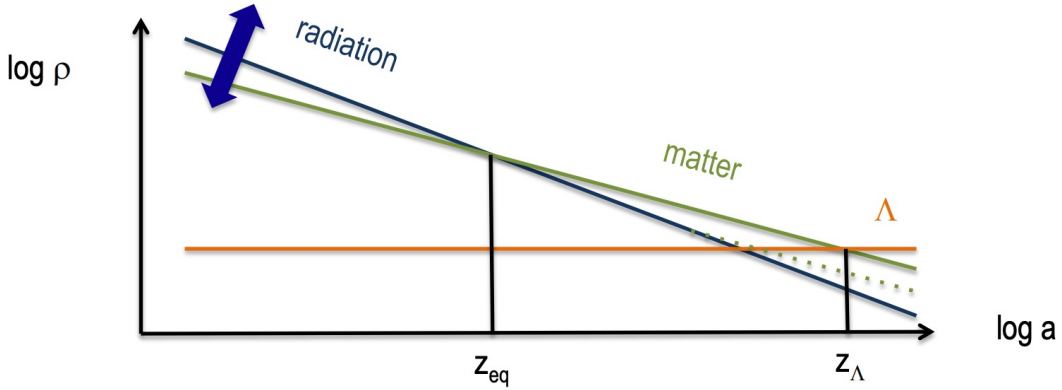
Aubourg+14



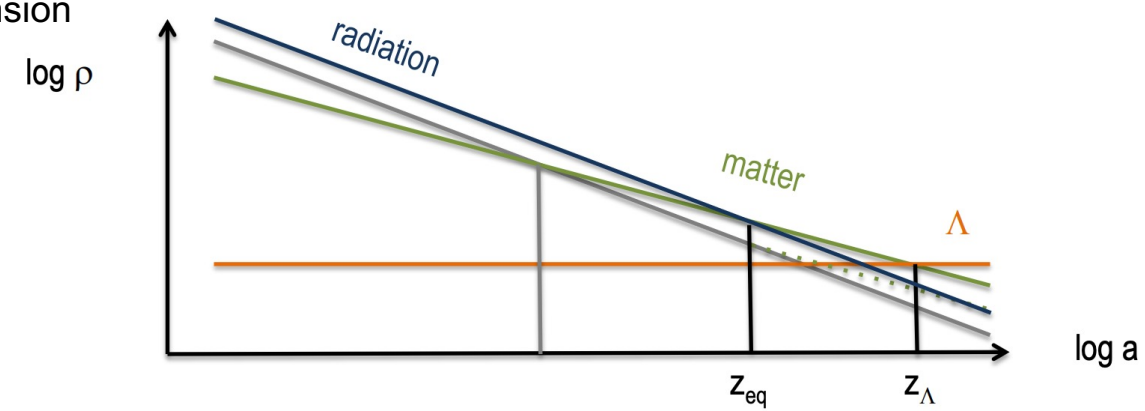
Neutrinos and BAOs

Matteo Viel

CMB
Constraints
Wcb independently
Of Wn



But late time expansion
And distances are
Affected by total
matter

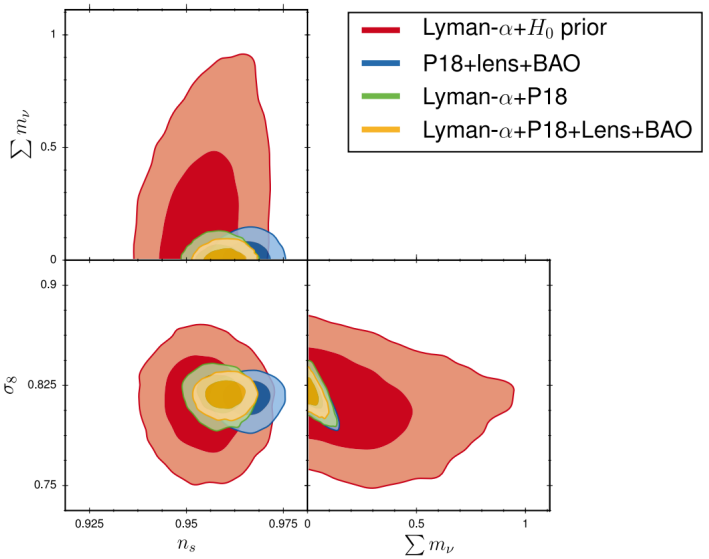


eBOSS data

	Frequentist		Bayesian	
	P18 + Lyman-α	P18 + Lyman-α +lens. +BAO	P18 + Lyman-α	P18 + Lyman-α +lens. +BAO
T_0 (z=3) (10^3K)	9.7 ± 1.7	9.8 ± 2.0	9.5 ± 1.8	9.5 ± 1.8
γ	0.69 ± 0.10	0.68 ± 0.11	0.71 ± 0.10	0.71 ± 0.10
σ_8	0.825 ± 0.006	0.819 ± 0.008	0.818 ± 0.010	0.818 ± 0.007
n_s	0.958 ± 0.003	0.961 ± 0.003	0.959 ± 0.004	0.960 ± 0.003
Ω_m	0.311 ± 0.006	0.308 ± 0.006	0.316 ± 0.009	0.310 ± 0.006
$\sum m_\nu$ (eV , 95% CL)	< 0.099	< 0.089	< 0.099	< 0.074

Table 6. Preferred astrophysical and cosmological parameter values (68.3% confidence level) for the ΛCDM + m_ν model, for combined Lyman-α, CMB and BAO data.

Palanque-Delabrouille+20



What to expect from DESI?

DESI and other Dark Energy experiments in the era of neutrino mass measurements

Andreu Font-Ribera[a],[1,2,†] Patrick McDonald,[2,‡] Nick Mostek,[2]
Beth A. Reid,[2,§] Hee-Jong Seo,[2,¶] and Anže Slosar[3,**]

[1] *Institute of Theoretical Physics, University of Zurich, Winterthurerstrasse 190, 8057 Zurich, Switzerland*
[2] *Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, USA*
[3] *Brookhaven National Laboratory, Upton, NY 11973, USA*

(Dated: May 16, 2014)

We present Fisher matrix projections for future cosmological parameter measurements, including neutrino masses, Dark Energy, curvature, modified gravity, the inflationary perturbation spectrum, non-Gaussianity, and dark radiation. We focus on DESI and generally redshift surveys (BOSS, HETDEX, eBOSS, Euclid, and WFIRST), but also include CMB (Planck) and weak gravitational lensing (DES and LSST) constraints. The goal is to present a consistent set of projections, for concrete experiments, which are otherwise scattered throughout many papers and proposals. We include neutrino mass as a free parameter in most projections, as it will inevitably be relevant – DESI and other experiments can measure the sum of neutrino masses to ~ 0.02 eV or better, while the minimum possible sum is ~ 0.06 eV. We note that constraints on Dark Energy are significantly degraded by the presence of neutrino mass uncertainty, especially when using galaxy clustering only as a probe of the BAO distance scale (because this introduces additional uncertainty in the background evolution after the CMB epoch). Using broadband galaxy power becomes relatively more powerful, and bigger gains are achieved by combining lensing survey constraints with redshift survey constraints. We do not try to be especially innovative, e.g., with complex treatments of potential systematic errors – these projections are intended as a straightforward baseline for comparison to more detailed analyses.

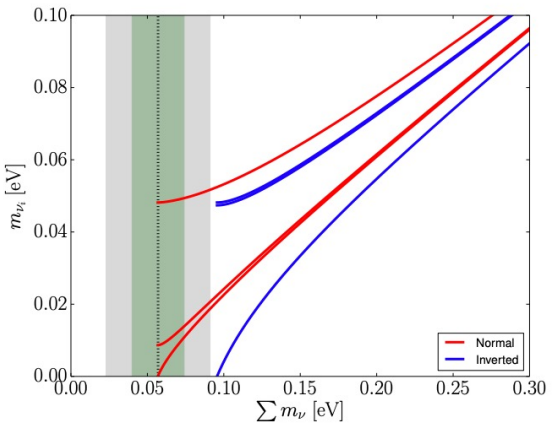


TABLE IX. Neutrino mass and other basic parameter projections. See Table VIII for experiment codes

	ω_m	ω_b	θ_s	Σm_ν	$\log_{10}(A)$	n_s	τ
value	0.141	0.0221	0.597	0.0600	−8.66	0.961	0.0920
P	0.0037	0.00015	0.00035	0.35	0.0039	0.0038	0.0045
$P + BgB + BlB$	0.00074	0.00015	0.00014	0.10	0.0038	0.0038	0.0044
$P + BgA0.1 + BlB$	0.00070	0.00013	0.00014	0.068	0.0037	0.0031	0.0044
$P + BgA0.2 + BlB$	0.00071	0.00012	0.00015	0.046	0.0037	0.0028	0.0043
$P + DES$	0.0013	0.00013	0.00017	0.041	0.0036	0.0032	0.0043
$P + BgB + BlB + DES$	0.00069	0.00011	0.00014	0.030	0.0035	0.0027	0.0043
$P + BgA0.1 + BlB + DES$	0.00067	0.00011	0.00014	0.029	0.0035	0.0027	0.0042
$P + BgA0.1 + BlB + ebA0.1$	0.00064	0.00012	0.00014	0.052	0.0037	0.0029	0.0043
$P + BgA0.2 + BlB + ebA0.2$	0.00064	0.00011	0.00014	0.036	0.0037	0.0027	0.0043
$P + BgA0.1 + BlB + ebA0.1 + DES$	0.00062	0.00011	0.00014	0.028	0.0035	0.0026	0.0042
$P + hdB + BgB$	0.00074	0.00015	0.00014	0.099	0.0038	0.0038	0.0044
$P + hdA0.1 + BgA0.1$	0.00069	0.00012	0.00014	0.061	0.0037	0.0030	0.0044
$P + hdA0.2 + BgA0.2$	0.00068	0.00011	0.00014	0.039	0.0037	0.0027	0.0043
$P + BBgB$	0.00055	0.00015	0.00014	0.090	0.0038	0.0038	0.0044
$P + BBgB + BlB$	0.00055	0.00015	0.00014	0.090	0.0038	0.0038	0.0044
$P + BBlB + BgB$	0.00072	0.00015	0.00014	0.098	0.0038	0.0038	0.0044
$P + BBgB + BBlB$	0.00055	0.00015	0.00014	0.090	0.0038	0.0038	0.0044
$P + BBgB + BBlB + DES$	0.00045	0.00011	0.00014	0.027	0.0035	0.0025	0.0043
$P + BBgA0.1$	0.00044	0.00011	0.00014	0.024	0.0036	0.0024	0.0043
$P + BBgA0.1 + BBlB$	0.00044	0.00011	0.00014	0.024	0.0036	0.0024	0.0043
$P + BBgA0.1 + BBlB + DES$	0.00043	0.00011	0.00014	0.021	0.0034	0.0024	0.0041
$P + BBgA0.2 + BBlB$	0.00042	0.00010	0.00014	0.017	0.0035	0.0022	0.0043
$P + BBgA0.2 + BBlB + DES$	0.00042	0.00010	0.00014	0.017	0.0033	0.0022	0.0040
$P + BB24gB + BB24lB$	0.00052	0.00015	0.00014	0.088	0.0038	0.0037	0.0044
$P + BB24gA0.1 + BB24lB$	0.00039	0.00011	0.00014	0.020	0.0035	0.0023	0.0043
$P + BB24gA0.1 + BB24lB + DES$	0.00038	0.00011	0.00013	0.019	0.0033	0.0023	0.0040
$P + BB24gA0.2 + BB24lB$	0.00037	$9.9e-05$	0.00014	0.015	0.0035	0.0020	0.0042
$P + BB24gA0.2 + BB24lB + DES$	0.00037	$9.9e-05$	0.00013	0.015	0.0032	0.0020	0.0040
$P + BgB + BlB + euB$	0.00054	0.00015	0.00014	0.090	0.0038	0.0038	0.0044
$P + BgA0.1 + BlB + euA0.1$	0.00043	0.00011	0.00014	0.021	0.0036	0.0024	0.0043
$P + BgA0.1 + BlB + euA0.1 + DES$	0.00043	0.00011	0.00014	0.019	0.0034	0.0023	0.0041
$P + BgA0.2 + BlB + euA0.2$	0.00042	0.00010	0.00014	0.015	0.0035	0.0021	0.0042
$P + BgA0.2 + BlB + euA0.2 + DES$	0.00041	0.00010	0.00014	0.015	0.0033	0.0021	0.0040
$P + BB24gA0.1 + BB24lB + euA0.1$	0.00036	0.00010	0.00013	0.017	0.0035	0.0022	0.0042
$P + BB24gA0.1 + BB24lB + euA0.1 + DES$	0.00036	0.00010	0.00013	0.016	0.0032	0.0022	0.0040
$P + BB24gA0.2 + BB24lB + euA0.2$	0.00034	$9.6e-05$	0.00013	0.014	0.0034	0.0018	0.0041
$P + BB24gA0.2 + BB24lB + euA0.2 + DES$	0.00034	$9.6e-05$	0.00013	0.013	0.0032	0.0018	0.0039
$P + LSST$	0.00080	0.00011	0.00015	0.020	0.0030	0.0029	0.0036
$P + BgB + BlB + LSST$	0.00060	0.00011	0.00014	0.018	0.0030	0.0025	0.0036
$P + BBgB + BBlB + LSST$	0.00044	0.00011	0.00013	0.016	0.0030	0.0022	0.0036
$P + BBgA0.1 + BBlB + LSST$	0.00042	0.00010	0.00013	0.015	0.0028	0.0021	0.0034
$P + BBgA0.2 + BBlB + LSST$	0.00041	0.00010	0.00013	0.014	0.0026	0.0020	0.0032
$P + BB24gA0.1 + BB24lB + LSST$	0.00038	0.00010	0.00013	0.015	0.0027	0.0020	0.0033
$P + BB24gA0.2 + BB24lB + LSST$	0.00036	$9.8e-05$	0.00013	0.013	0.0025	0.0018	0.0031
$P + BB24gA0.1 + BB24lB + euA0.1 + LSST$	0.00035	0.00010	0.00013	0.014	0.0026	0.0019	0.0032
$P + BB24gA0.2 + BB24lB + euA0.2 + LSST$	0.00033	$9.5e-05$	0.00013	0.011	0.0024	0.0016	0.0030
$P + wfB + BgB$	0.00064	0.00015	0.00014	0.095	0.0038	0.0038	0.0044
$P + wfA0.1 + BgA0.1$	0.00058	0.00011	0.00014	0.037	0.0037	0.0027	0.0043
$P + wfA0.2 + BgA0.2$	0.00056	0.00011	0.00014	0.021	0.0036	0.0025	0.0043
$P + BgB + BlA + l1D$	0.00066	0.00011	0.00014	0.053	0.0037	0.0032	0.0044
$P + BgA0.1 + BlA + l1D$	0.00065	0.00011	0.00014	0.048	0.0037	0.0030	0.0043
$P + BgA0.2 + BlA + l1D$	0.00066	0.00011	0.00014	0.040	0.0037	0.0027	0.0043
$P + BBgB + BBlA + l1D$	0.00041	0.00010	0.00014	0.039	0.0037	0.0029	0.0043
$P + BBgA0.1 + BBlA + l1D$	0.00039	0.00010	0.00014	0.023	0.0035	0.0021	0.0043
$P + BBgA0.2 + BBlA + l1D$	0.00038	0.00010	0.00014	0.017	0.0035	0.0019	0.0042
$P + BB24gB + BB24lA + l1D$	0.00036	0.00010	0.00014	0.034	0.0036	0.0028	0.0043
$P + BB24gA0.1 + BB24lA + l1D$	0.00035	0.00010	0.00013	0.019	0.0035	0.0019	0.0042
$P + BB24gA0.2 + BB24lA + l1D$	0.00034	$9.8e-05$	0.00014	0.015	0.0034	0.0016	0.0041
$P + BB24gA0.2 + BB24lA + l1D + euA0.2$	0.00032	$9.5e-05$	0.00013	0.013	0.0033	0.0015	0.0040
$P + BB24gA0.2 + BB24lA + l1D + LSST$	0.00033	$9.7e-05$	0.00013	0.012	0.0025	0.0015	0.0031
$P + BB24gA0.2 + BB24lA + l1D + euA0.2 + LSST$	0.00032	$9.5e-05$	0.00013	0.011	0.0024	0.0014	0.0030

Matter power spectrum and WDM

2 well studied examples:
Particles becoming NR in RD era → free streaming scale is constant between zNR and zEQ but Free streaming horizon grows!
1) thermal WDM with FD distribution and unknown Tx temperature
2) non-thermal relic with rescaled FD distribution and Tx=Tν and some unkonw rescaling factor χ
Like Dodelson&Widrow (94) sterile non-resonantly produced neutrinos

$$(m_{th}, T_{th}) = (\chi^{1/4} m_X, \chi^{1/4} T_\nu)$$

$$P_{\Lambda WDM}(\eta, k) = P_{\Lambda CDM}(\eta, k) T(\eta, k)^2$$

Sterile neutrinos Early decoupled thermal relics

$$\omega_X = \chi (T_\nu / T_\nu^a)^3 \frac{m_X}{94.1 eV} = (T_{th}/T_\nu^a)^3 \frac{m_{th}}{94.1 eV}$$

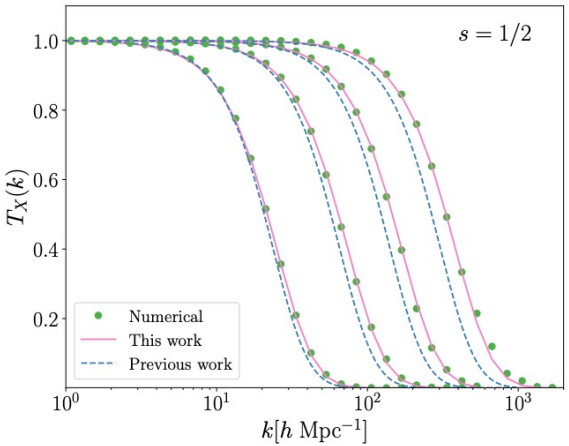
$$m_X = 4.43\, keV (T_\nu / T_\nu^a)(0.25 \times 0.7^2/\omega_X)^{1/3} \left(\frac{m_{th}}{1 keV}\right)^{4/3}$$

Matter power spectrum and WDM

$$T(k) \equiv [1 + (k/k_{break})^p]^{-10/p} \quad with\, p = 2.24$$

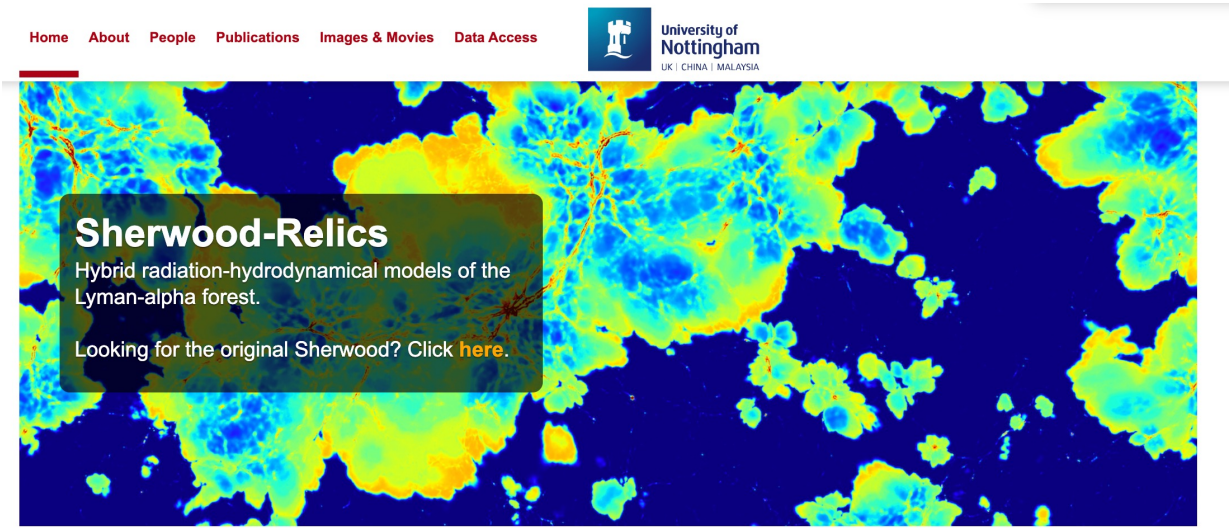
$$k_{break} = \frac{1}{0.24} X^{0.83} \left(\frac{\omega_X}{0.25 \times 0.7^2} \right)^{0.16} Mpc^{-1} \quad with\, X \equiv \frac{m_X/T_X}{1\, keV} T_\nu^a$$

Important: unlike active neutrinos this depends on both DM density and X
Because free streaming horizon depends on those



Viel+05;
Vogel&Abazajian https://arxiv.org/abs/2210.10753

Sherwood-Relics Collaboration



People

- Shikhar Asthana
- James Bolton
- Emma Chapman
- Prakash Gaikwad
- Martin Haehnelt
- Vid Irsic
- Laura Keating
- Girish Kulkarni
- Ewald Puchwein (PI)
- Tomáš Šoltinský
- Matteo Viel

Physics of the Lyα forest - I

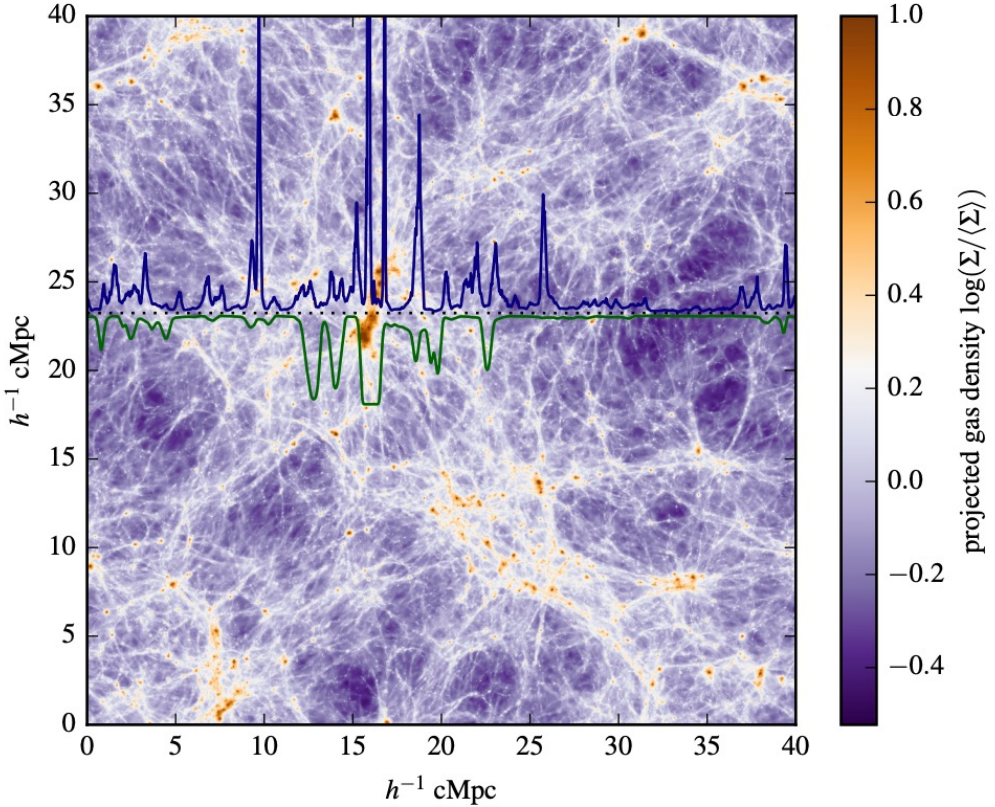
Matteo Viel

Relatively "simple" physics

- Gravity
- Cooling/Heating processes
- Few runs with feedback (winds/AGN)
- Radiative transfers due to patchy reionization included

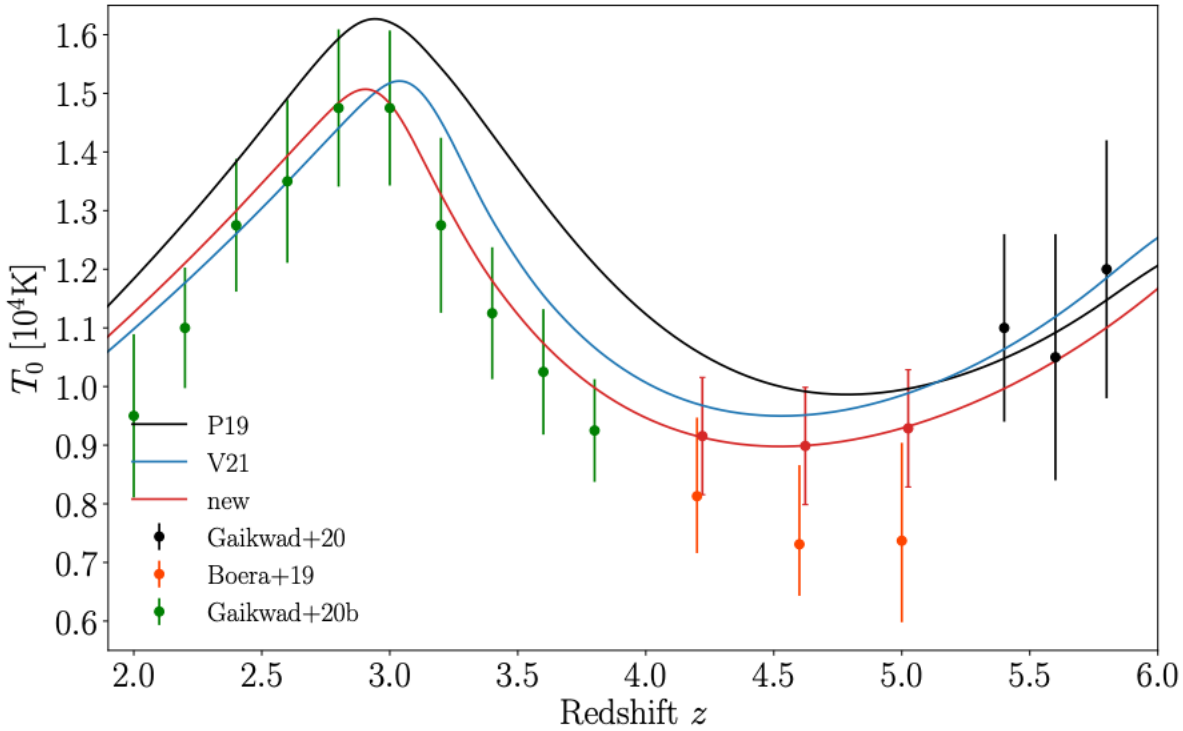
Gadget-III + ATON code
About 400 simulations
Post-process to provide >60,000 flux 1D power models (varying also cosmology trough slope and amplitude of linear matter power)
About 75 Million CPU hrs
Boxes 5-160 Mpc/h
Resolutions 10^3-10^6 Msun/h per gas
Reference sims have 40 Mpc/h
and 10^5 Msun/h per gas

High-z (2<z<6) cosmic web
Sherwood simulation suite – Bolton+17



Physics of the Lyα forest - III

Matteo Viel



IGM thermal state

Constraints obtained with a huge variety of data and methods

Sensitive to lines rather than the clustering of the lines

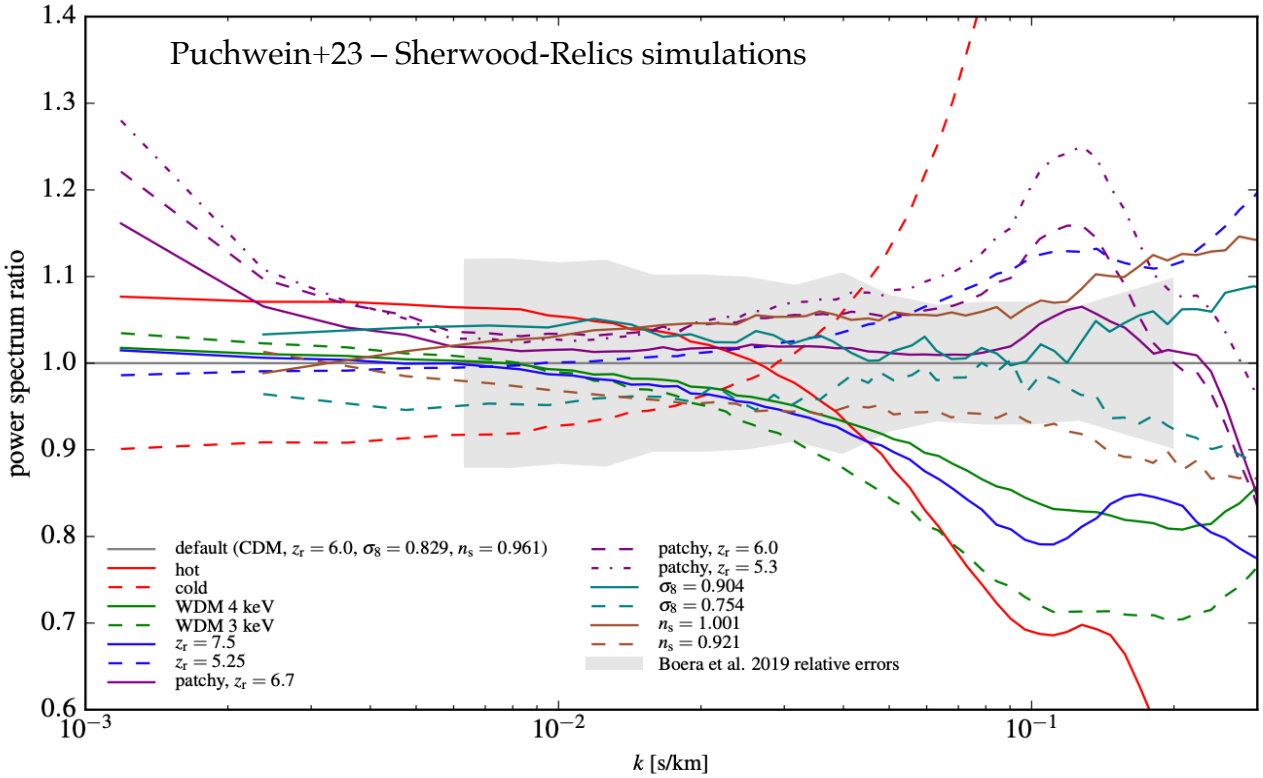
HeII bump quite well detected

Physics of the Lyα forest - IV

Matteo Viel

- Large scale increase due to patchy reionization
- Small scale increase/decrease due to WDM/temperature
- Intermediate regime also quite constraining

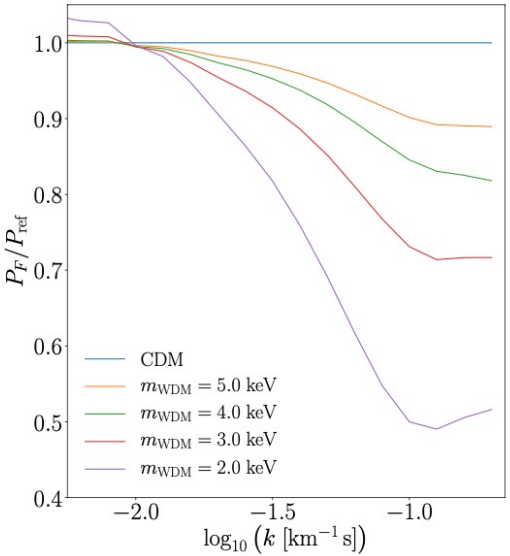
Simulated 1D flux power @ z=4.6



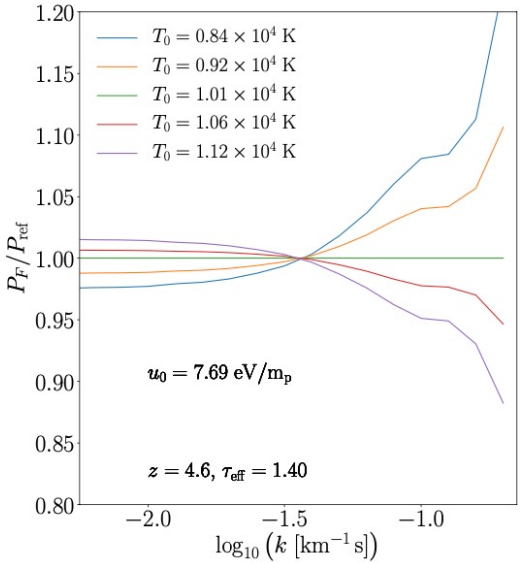
Physics of the Lyα forest - V

Matteo Viel

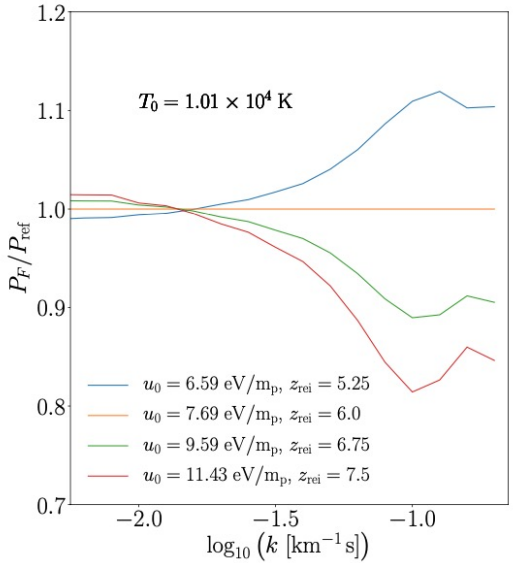
WDM free streaming



Thermal broadening



Gas pressure

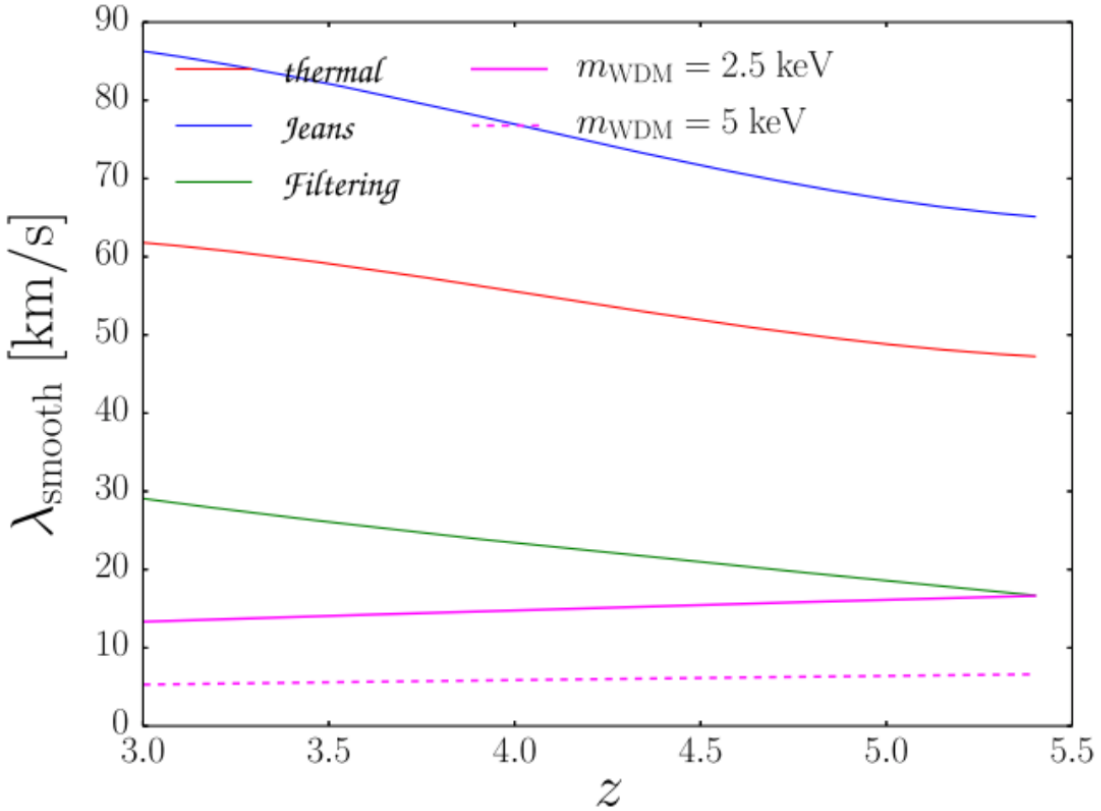


$$u_0(t) = \int_0^t dt \frac{\mathcal{H}}{\bar{\rho}_m} \frac{3k_B}{2\mu}$$

$\mathcal{H}$ is heating rate

Physics of the Lyα forest - VI

Matteo Viel



- Different physical scales (on top of instrumental resolution) affect the power spectrum cutoff:

- thermal: instataneous temperature at that redshift;

- Jeans: scale due to gas pressure;

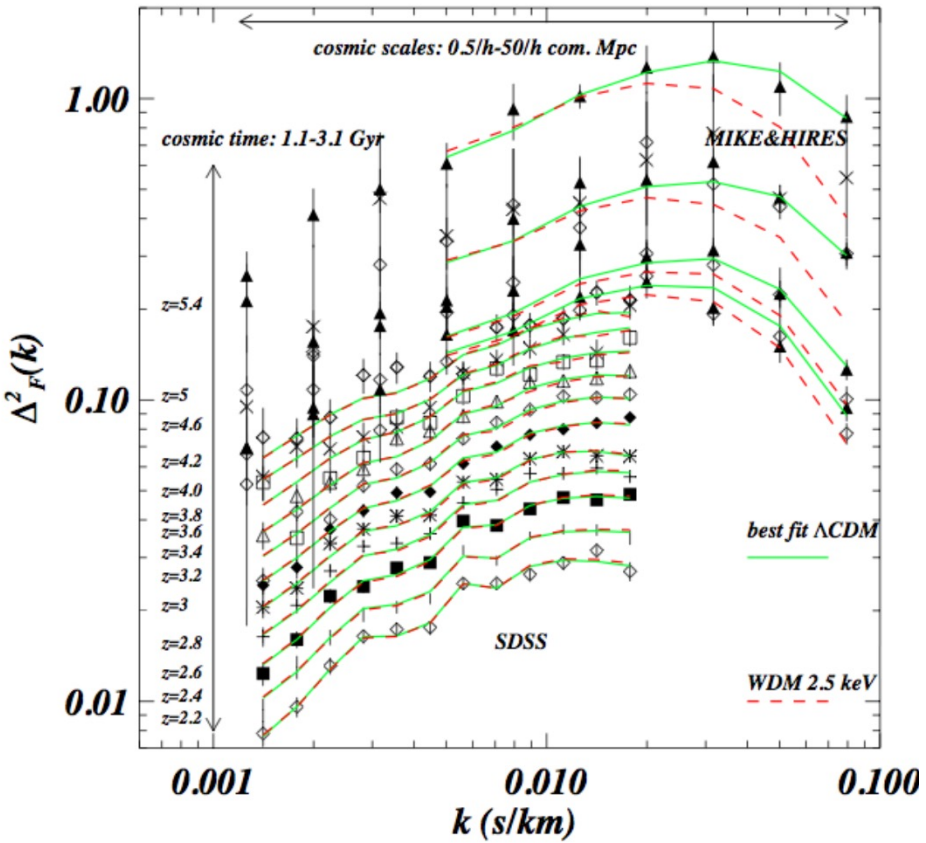
- filtering scale: depends on all the past thermal history;

- WDM cutoffs are basically redshift independent

- Constraints are obtained from **a full shape** of the 1D fux power.

Status in 2013

Viel+13



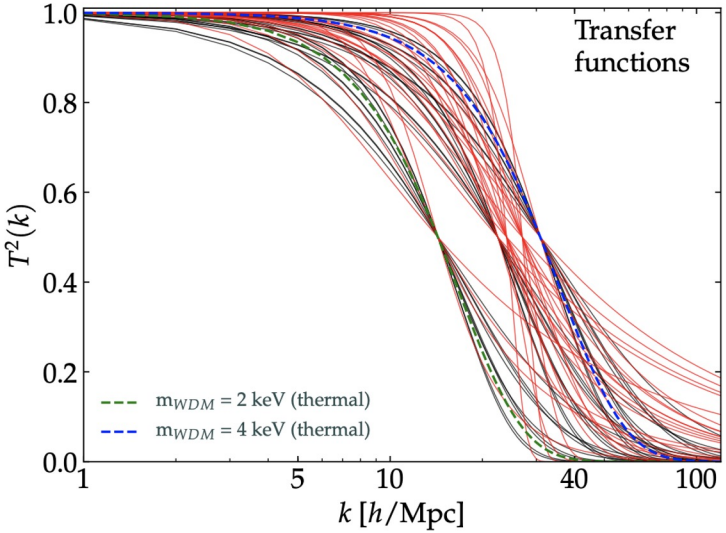
- Test of structure formation for a LCDM Universe in a **unique "pre-galactic" environment**

- $m_{WDM} > 3.3$ keV (2σ C.L.)

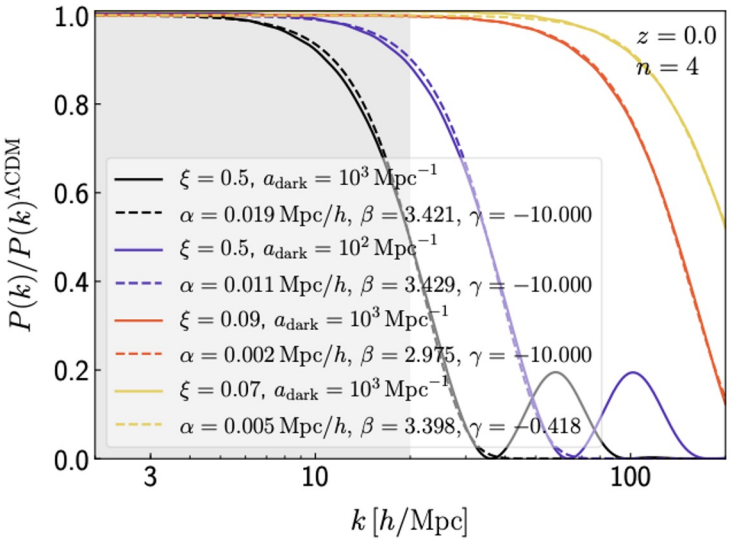
Note: 10 yrs later only a factor 2 more high-z QSOs

"New" non-cold Dark Matter models

Matteo Viel



Murgia+17



Archidiacono+19

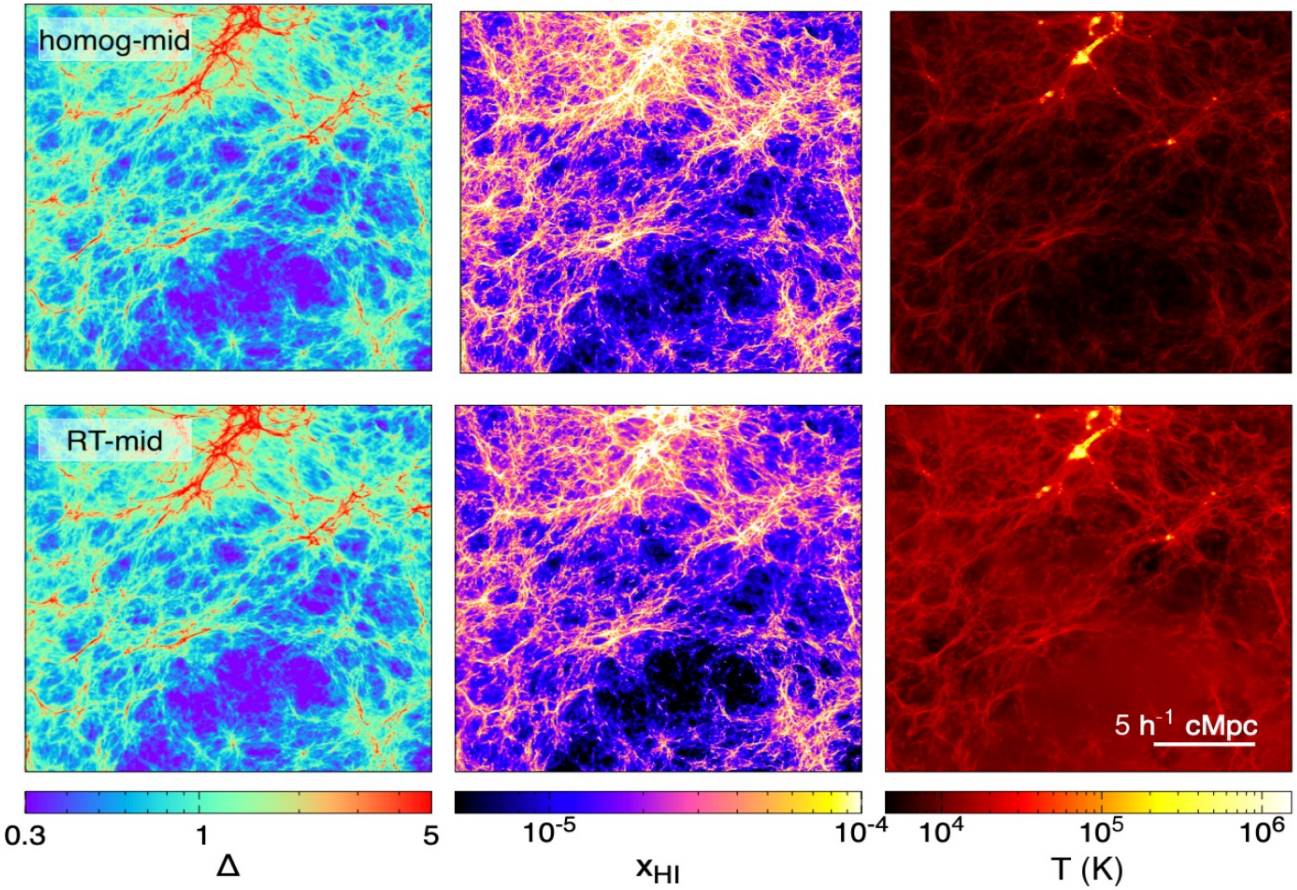
Features to be constrained: shape (before and after $k_{1/2}$) - plateau - Oscillations/bumps in power

Patchy Reionization - I

Matteo Viel

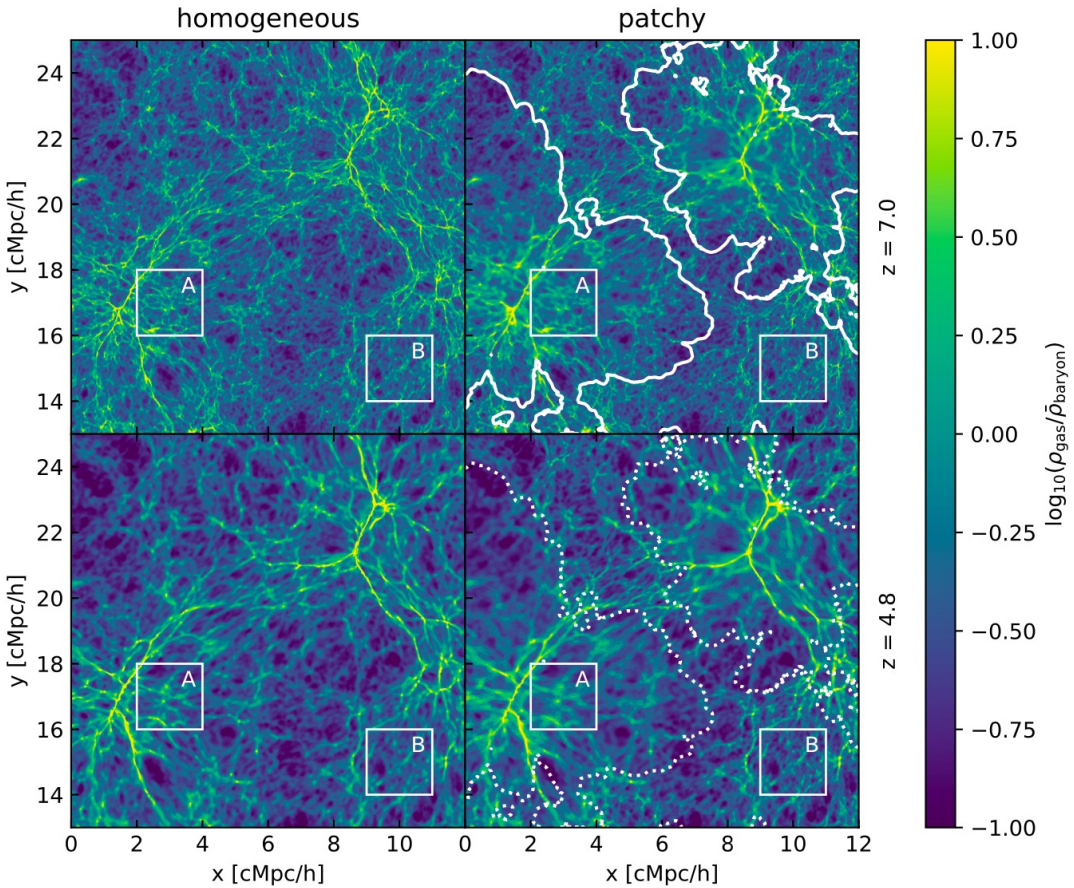
Molaro, Irsic+21, +23

z =5.4



Patchy Reionization - II

Matteo Viel



Puchwein+23

During reionization

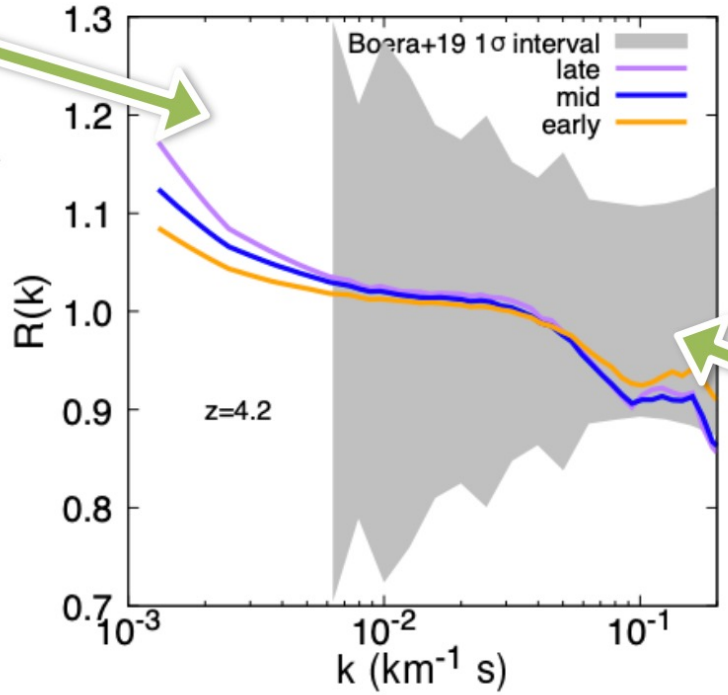
Note:
Reionization ends
at z =5.4

After reionization is complete

Patchy Reionization - III

Matteo Viel

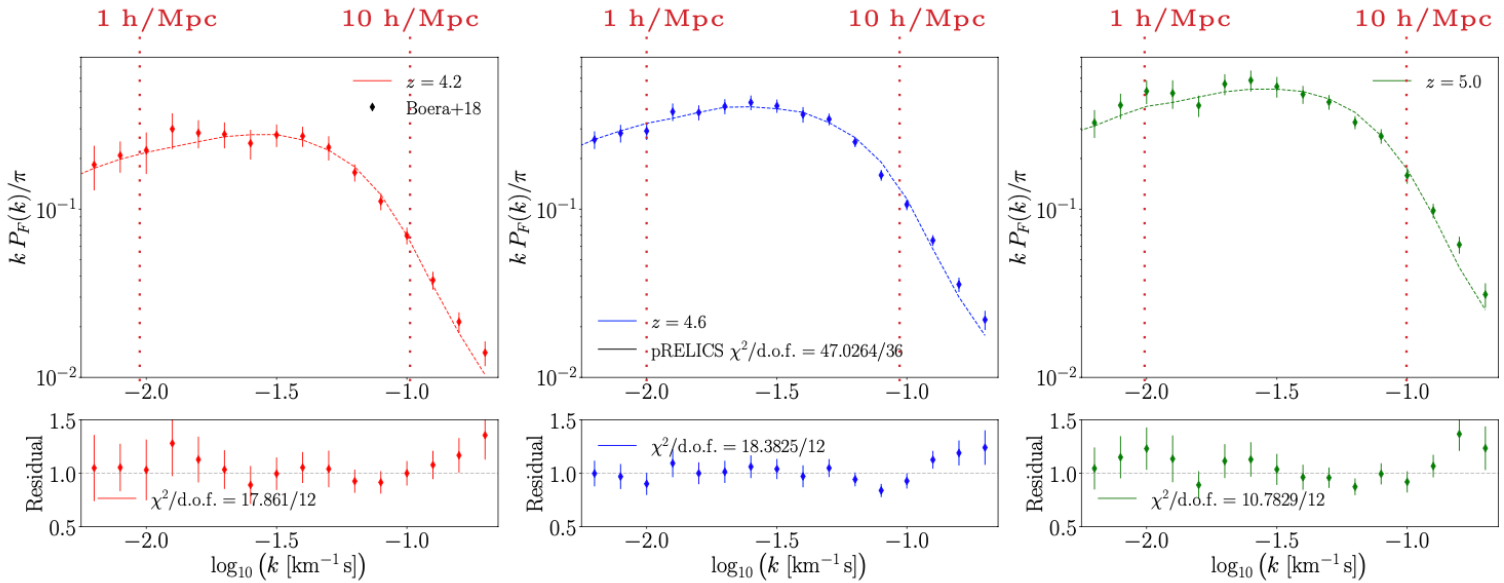
Boost
due to variation
in neutral fraction
from large scale
temperature fluctuations



Suppression
due to thermal broadening
of recently ionised regions

Molaro, Irsic+21, +23

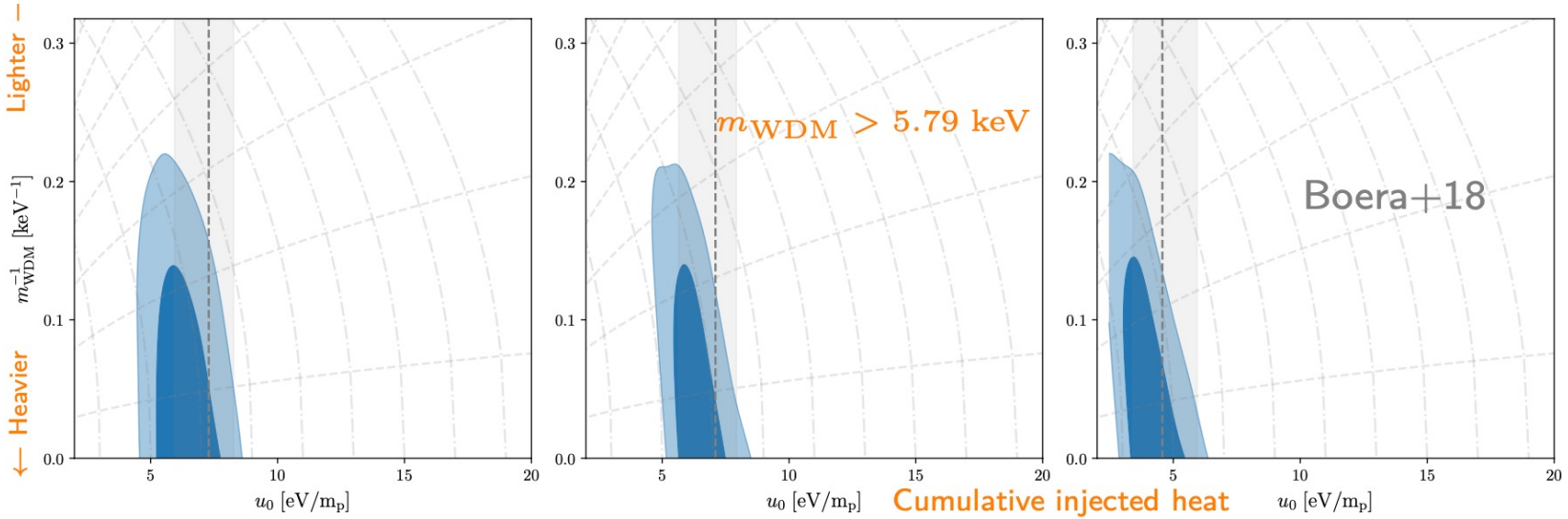
Thermal WDM - III



Irsic+23

Thermal WDM - III

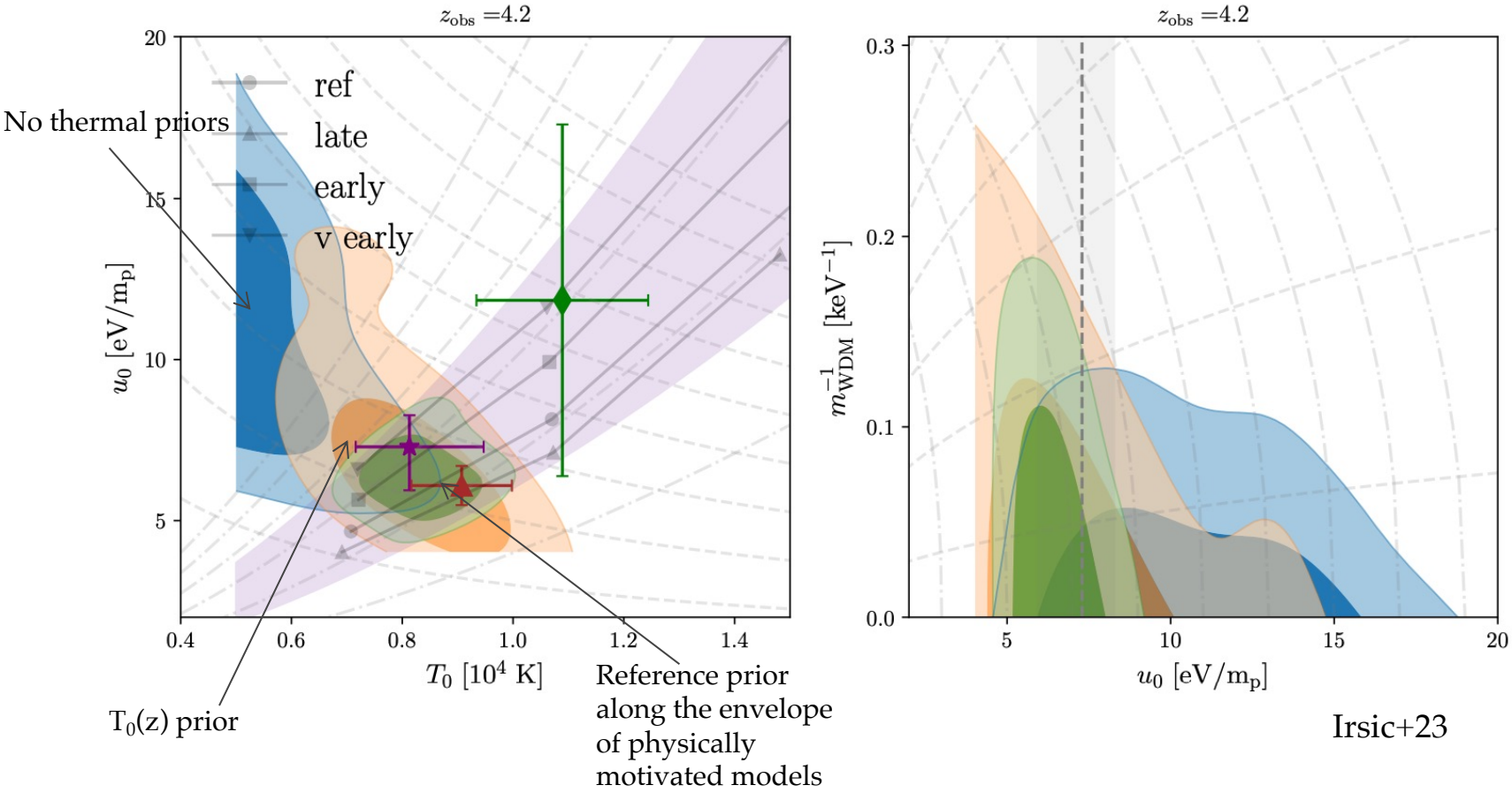
Matteo Viel



Irsic+23

Thermal WDM – the effect of thermal priors

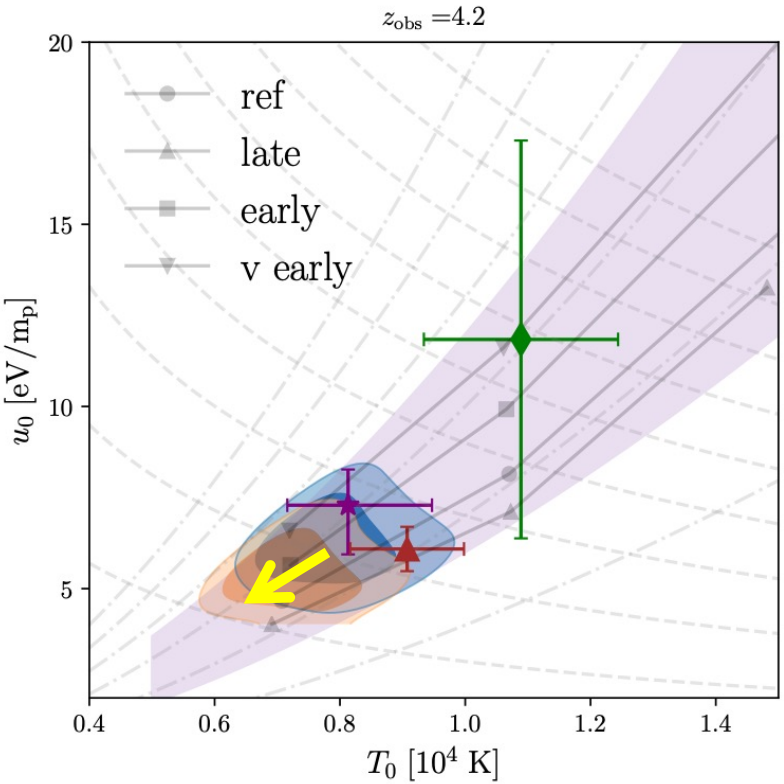
Matteo Viel

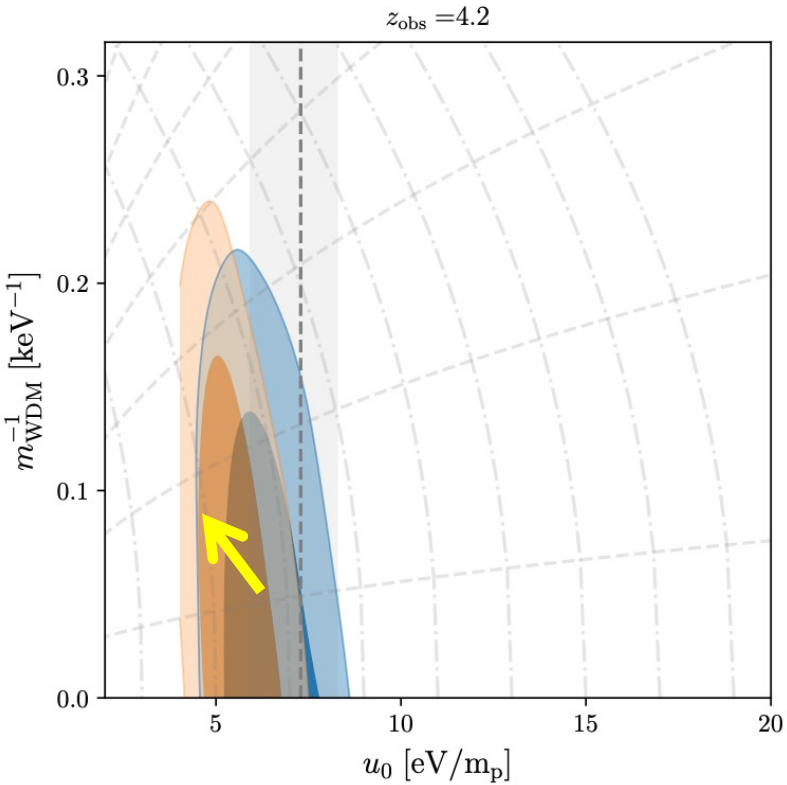


Irsic+23

Thermal WDM – inclusion of patchy correction

Matteo Viel

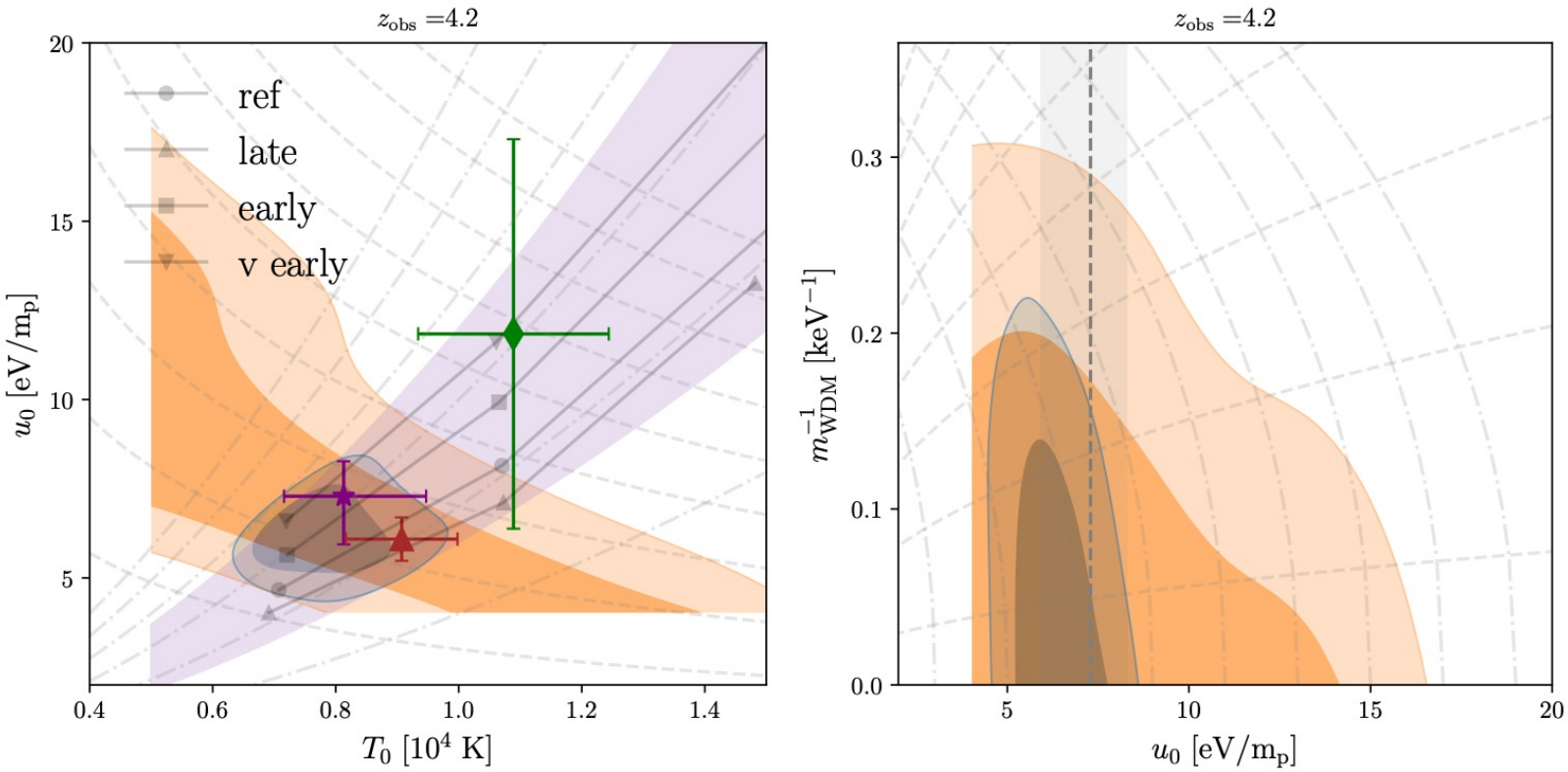




Irsic+23

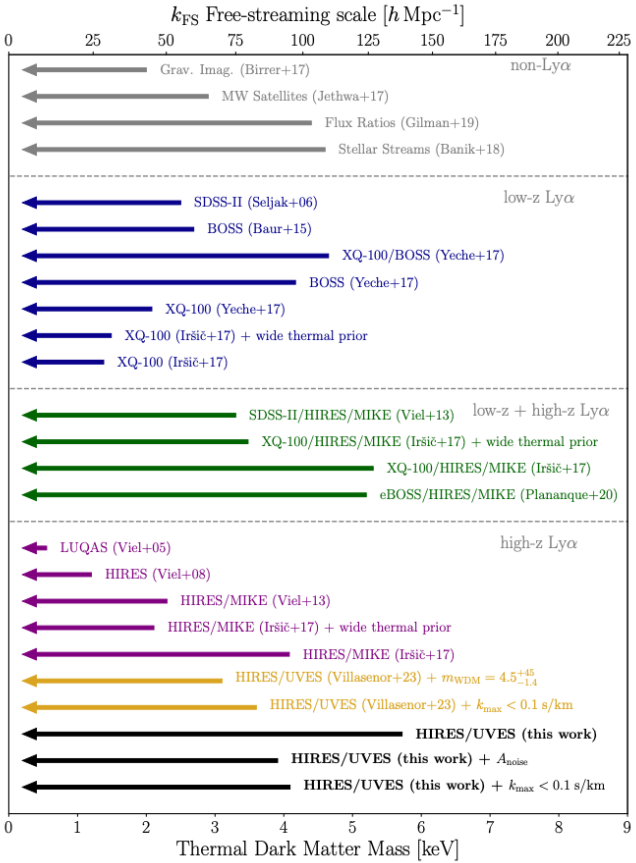
Thermal WDM – noise correction

Matteo Viel



Irsic+23

Thermal WDM - III



Name	$m_{\rm WDM}$ [keV] (2σ)	$\tau_{\rm eff}(z=4.6)$	$T_0(z=4.6)$ [10^4 K]	$\gamma(z=4.6)$	$u_0(z=4.6)$ [eV/m$_{\rm p}$]	$A_{\rm noise}(z=4.6)$	χ^2/dof
Default	> 5.72	$1.502^{+0.061}_{-0.061}$	$0.743^{+0.041}_{-0.075}$	$1.35^{+0.24}_{-0.19}$	$6.19^{+0.68}_{-0.68}$	-	48.9/34
$k_{\rm max} < 0.1$ km^{-1} s	> 4.10	$1.501^{+0.060}_{-0.074}$	$0.840^{+0.095}_{-0.340}$	$1.28^{+0.09}_{-0.28}$	$8.91^{+1.57}_{-5.26}$	-	12.6/20
$A_{\rm noise}$	> 3.91	$1.458^{+0.053}_{-0.074}$	$0.966^{+0.156}_{-0.466}$	$1.23^{+0.06}_{-0.23}$	$5.93^{+0.38}_{-2.28}$	$1.20^{+0.49}_{-0.29}$	23.8/31
$T_0(z)$ prior	> 5.85	$1.494^{+0.062}_{-0.077}$	$0.770^{+0.110}_{-0.120}$	$1.31^{+0.10}_{-0.31}$	$6.50^{+1.00}_{-1.60}$	-	65.6/34
$R_s(u_0)$ mass resolution	> 4.44	$1.531^{+0.073}_{-0.064}$	$0.617^{+0.007}_{-0.118}$	$1.38^{+0.28}_{-0.13}$	$7.90^{+1.70}_{-2.30}$	-	29.6/34
patchy reion.	> 5.10	$1.486^{+0.058}_{-0.068}$	$0.686^{+0.046}_{-0.080}$	$1.33^{+0.17}_{-0.26}$	$5.32^{+0.58}_{-0.52}$	-	55.3/34

Irsic+23

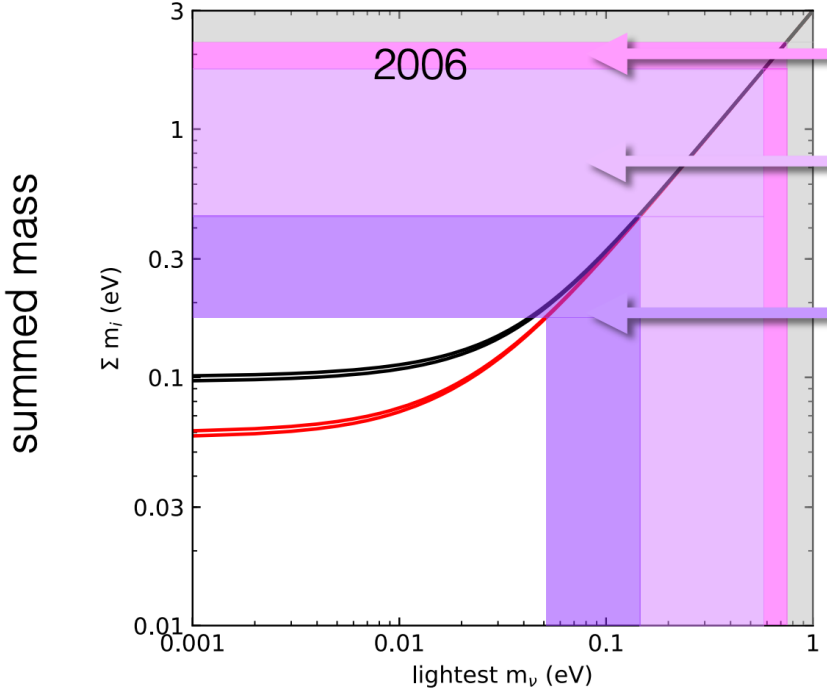
Plan

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Some historical background (a successful story!)

Matteo Viel

95%CL upper bounds on $\Sigma_i m_i$



CMB only : WMAP, VSA, ACBAR, CBI…

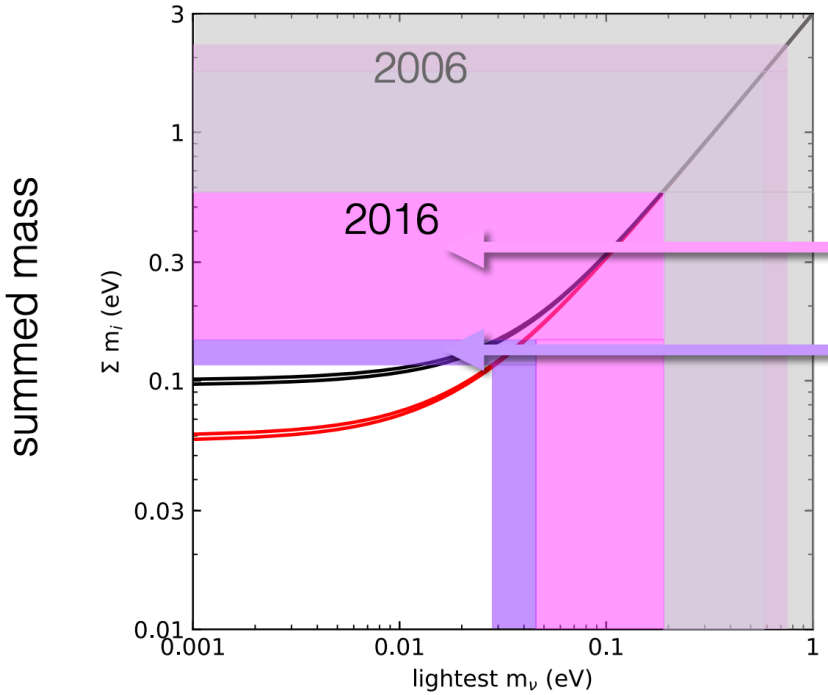
CMB + LSS : 2dF, SDSS-BAO, SDSS-power spectrum

CMB + LSS : Lyman-alpha

Seljak et al. 2006; Viel et al. 2006

Some historical background

95%CL upper bounds on $\Sigma_i m_i$



CMB only: Planck, w/o polarisation and lensing…
$\Sigma_i m_i$ < 590 to 140 meV (95%CL)

[Planck col.] 1605.02985

CMB + LSS :

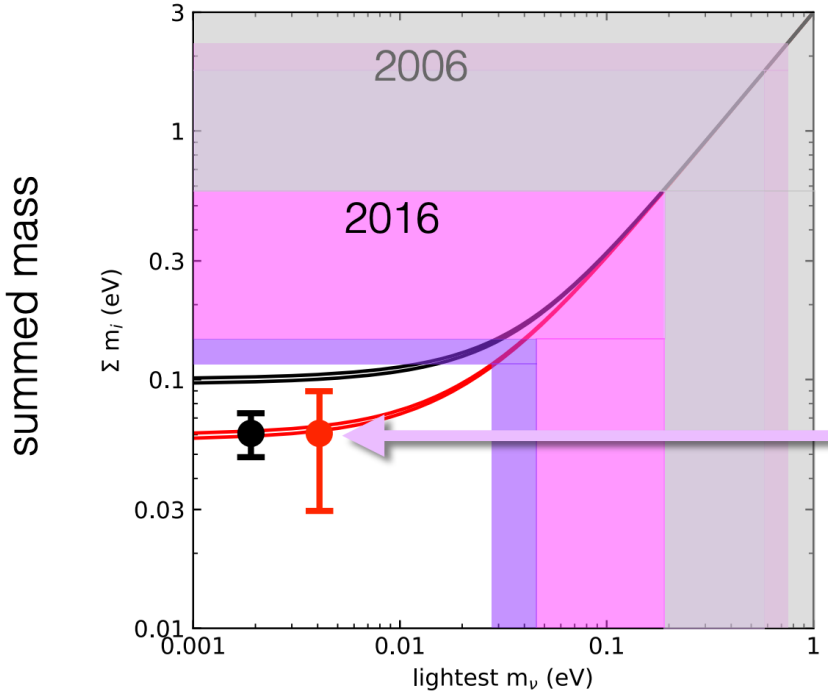
- with galaxies: $\Sigma_i m_i$ < 130 meV (95%CL)
- with Lyman-α: $\Sigma_i m_i$ < 120 meV (95%CL)
- with BAOs: $\Sigma_i m_i$ < 118 meV (95%CL)

Cuesta et al. 1511.05983; Palanque-Delabrouille et al. 1506.05976; Vagnozzy et al. 1701.08172

NH already favoured by cosmo. + labo. bounds (Simpson et al. 1703.03425; Capozzi et al. 1703.04471)

Some historical background

95%CL upper bounds / 1σ forecast errors on $\Sigma_i m_i$



bayer22.pdf

Planck + next generation LSS :
DES, DESI, Euclid, LSST, wFIRST, SKA

σ ~ { **40 -> 12 meV (7 params + …)**
60 -> 30 meV (complicated DE)
60 -> 40 meV (complicated MG)

e.g. Font-Ribera et al. 1308.4164

… with conservative use of SKA; 21cm?

New probes/new issues

- Baryons?
- Cosmic Voids
- 21cm cosmology
- Environmental effects
- Higher order?

Weak lensing

$$C^{(ij)}(\ell) = \int_0^\infty \mathrm{d}z \; \frac{c}{H(z)} \; \frac{W^{(i)}(z) \; W^{(j)}(z)}{\chi^2(z)} \; P_{\rm mm}\left(k = \frac{\ell}{\chi(z)}, z\right)$$

$$W^{(i)}(z) = \frac{3}{2} \; \Omega_{\rm m} \left(\frac{H_0}{c}\right)^2 (1+z) \; \chi(z) \int_{\min\,(z, z_i)}^{z_{i+1}} \mathrm{d}x \; n_{\rm s}(x) \; \frac{\chi(x) - \chi(z)}{\chi(x)}$$

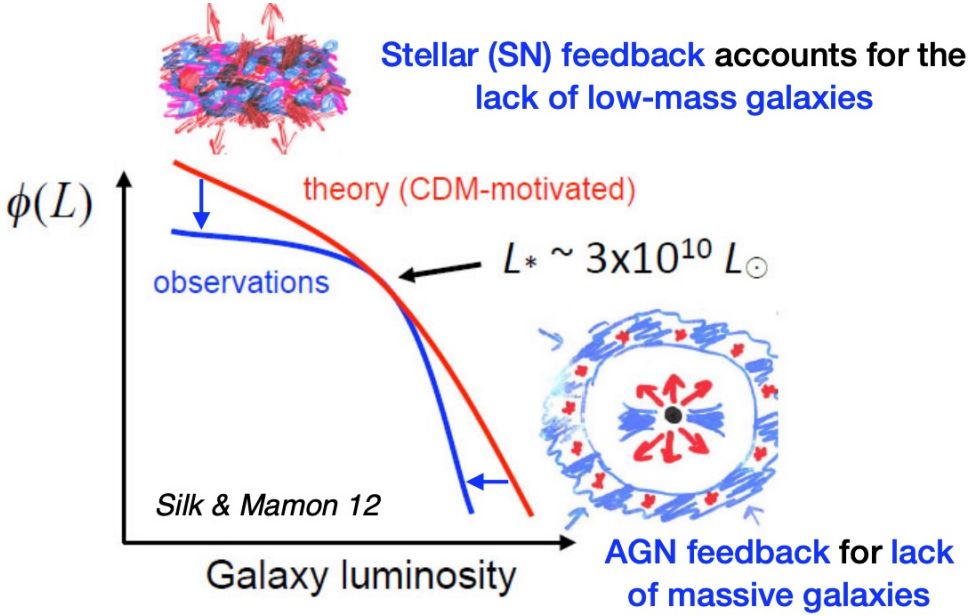
$$k_{\rm fs} = 0.82 \; \frac{E(z)}{(1+z)^2} \; \frac{M_\nu}{1 \; \mathrm{eV}} \, h \, \mathrm{Mpc}^{-1}$$

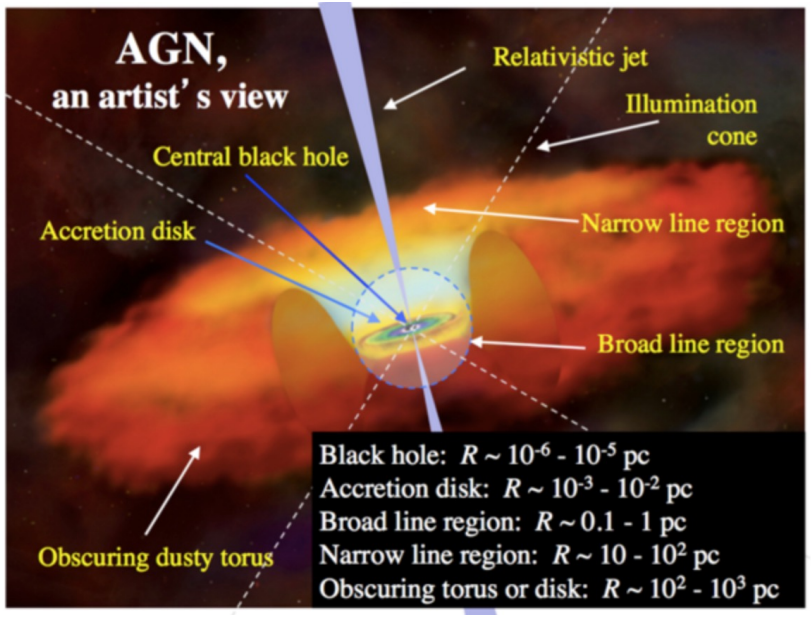
$$P_{\rm mm}(k) = (1 - f_\nu)^2 \, P_{\rm cc}(k) + 2 \, f_\nu \, (1 - f_\nu) \, P_{\rm c\nu}(k) + f_\nu^2 \, P_{\nu\nu}(k)$$

$$\frac{\Delta P^{\rm L}_{\rm cc}(k)}{P^{\rm L}_{\rm cc}(k)} \approx -6 \; f_\nu, \qquad \frac{\Delta P^{\rm L}_{\rm mm}(k)}{P^{\rm L}_{\rm mm}(k)} \approx -8 \; f_\nu$$

Baryon feedback - I

Matteo Viel





Baryon feedback - II

$$F_{\rm bf}(k,z|M_c,\eta_{\rm b},z_{\rm c}) \equiv \frac{P_{\rm feed}(k)}{P_{\rm dmo}(k)} = \left\{ \frac{B(z)}{1+(k/k_g)^3} + [1-B(z)] \right\} \; S(k),$$

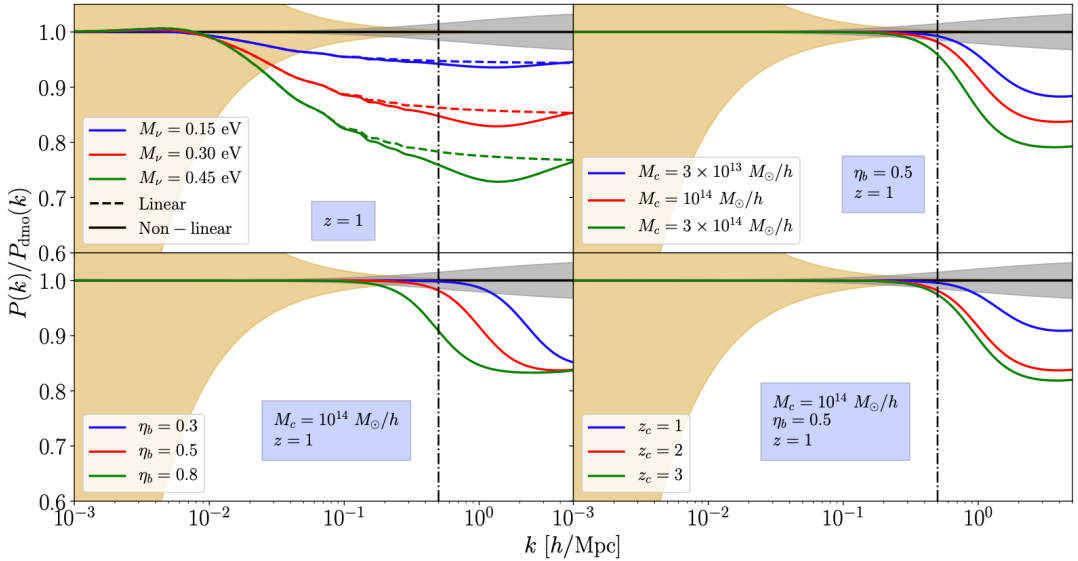
where

$$B(z) = \frac{0.105 \log\left(\frac{M_c}{{\rm M}_\odot/h}\right) - 1.27}{1+(z/z_{\rm c})^{2.5}},$$

$$S(k) = 1 + \left(\frac{k}{55\,h\,{\rm Mpc}^{-1}}\right)^2$$

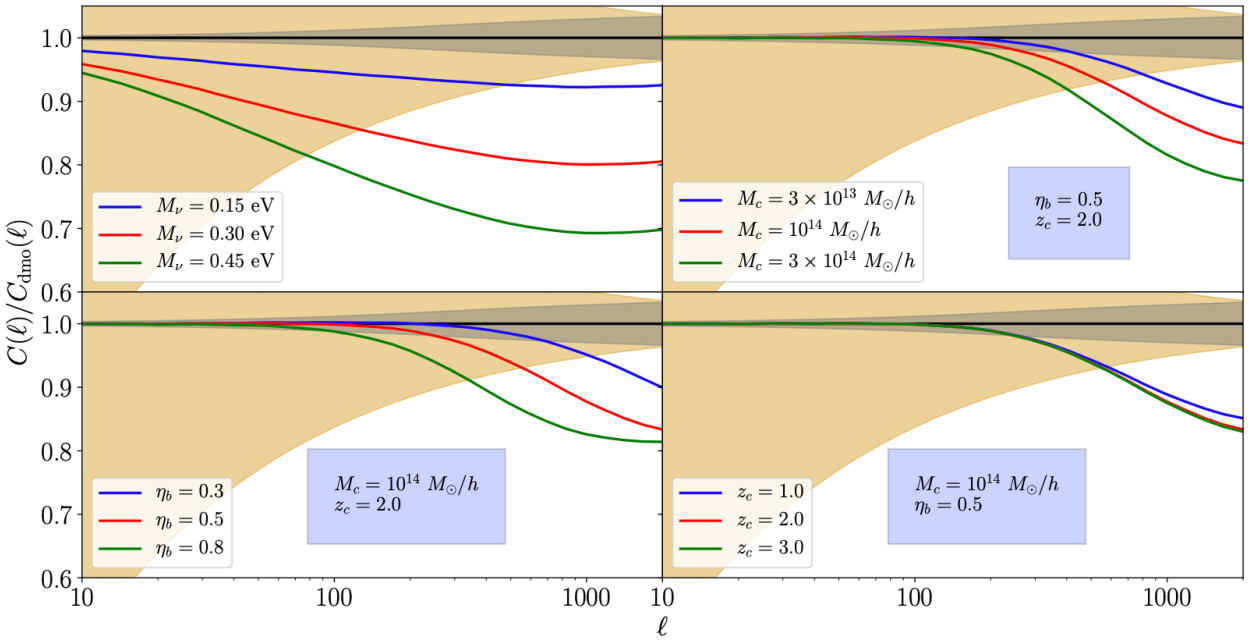
for $M_c \geq 10^{12}$ ${\rm M}_\odot/h$ and zero otherwise,

$$k_g(z) = 0.7\;[1-B(z)]^4\;\eta_{\rm b}^{-1.6}\;h\,{\rm Mpc}^{-1},$$



Baryon feedback - III

Matteo Viel



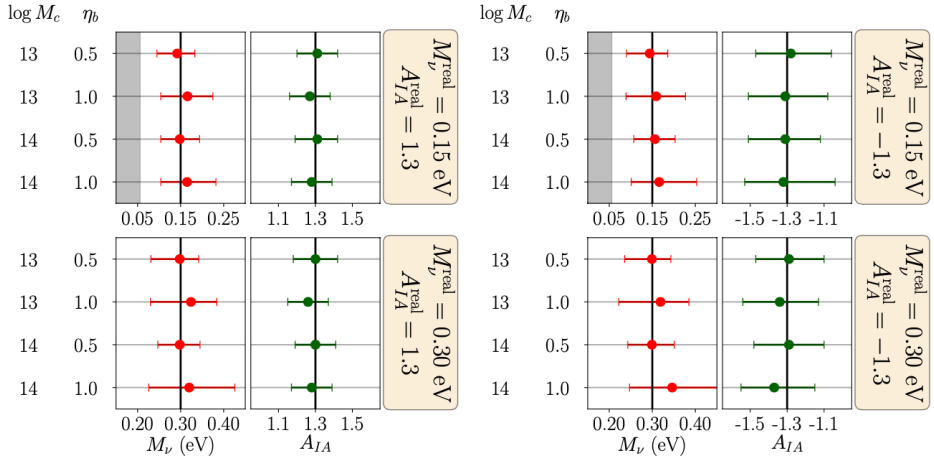
Baryon feedback - IV

Maximum shift in the total neutrino mass is 0.5s

No obvious degeneracies

Apart from one: between neutrino mass and intrinsic alignment (important term in weak lensing modelling)

In general: neutrino free streaming is more gentle, and with different z-dependence compared to baryon feedback

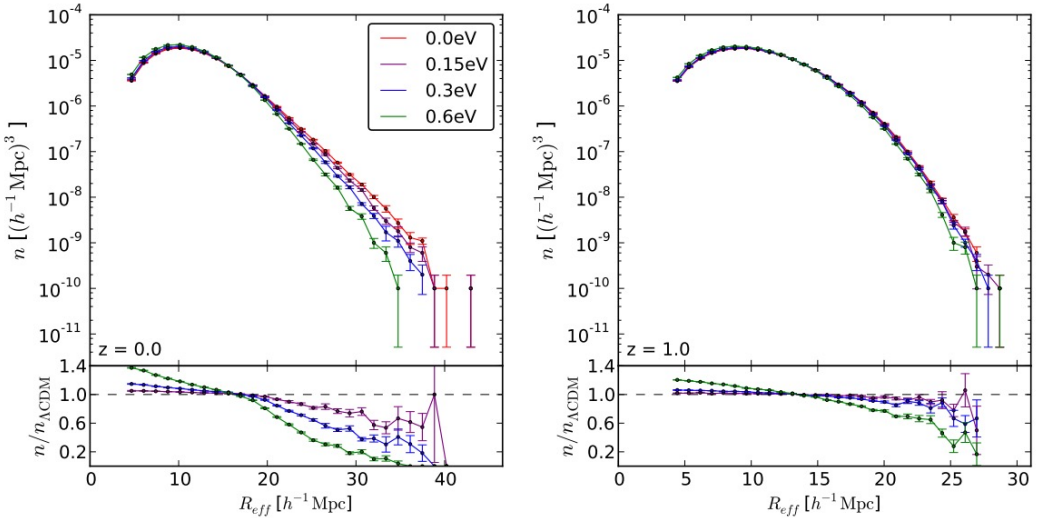


Voids: the void size function

Voids in massive neutrino cosmologies

Elena Massara,[a,b] Francisco Villaescusa-Navarro,[c,b] Matteo Viel,[c,b] P. M. Sutter [c,b,d]

Comprehensive numerical effort



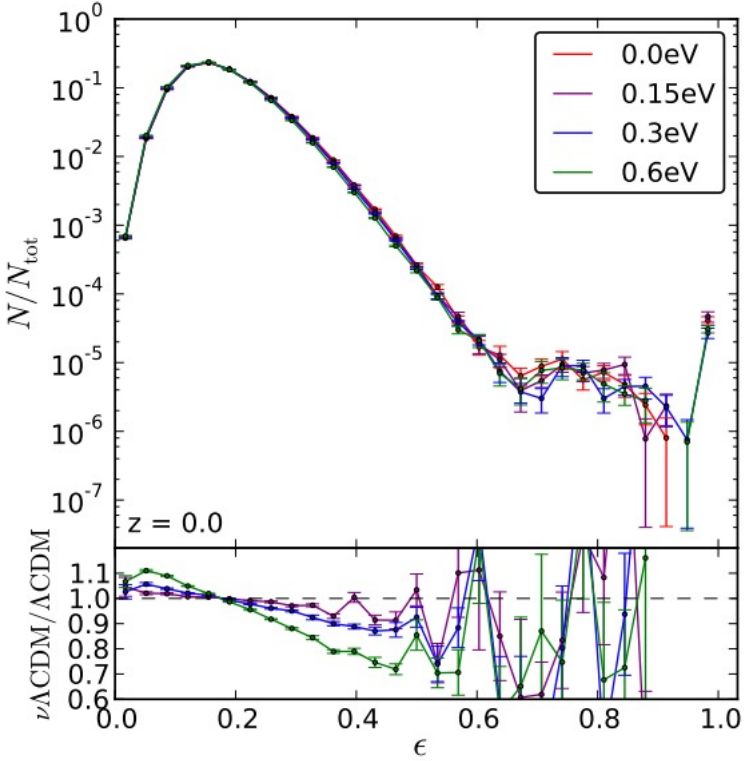
Voids in massive neutrino cosmologies are less evolved (i.e. **younger**) than those in the corresponding massless neutrinos case: there is a larger number of small voids and a smaller number of large ones, their profiles are less evacuated, and they present a lower wall at the edge.

VOIDS evolve by **evacuating particles** (very different from haloes)!

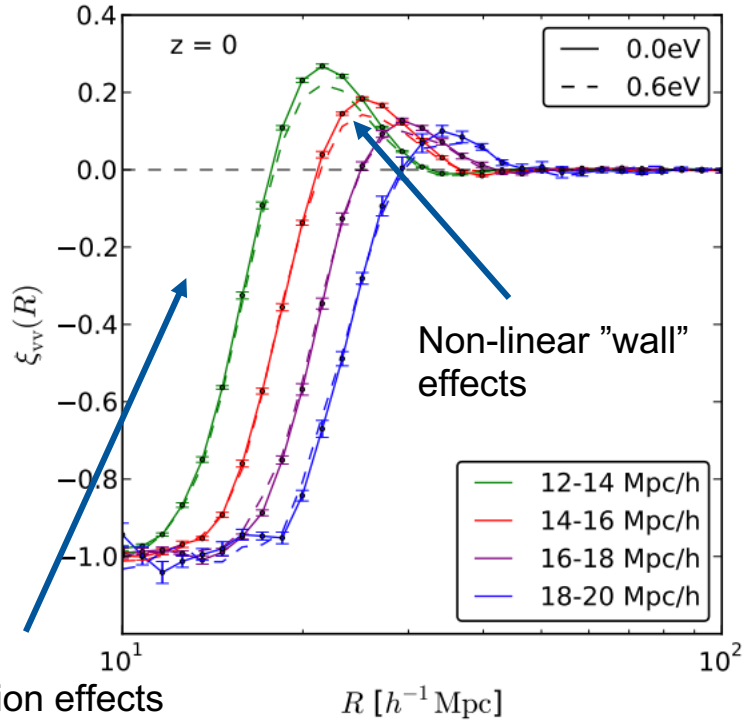
Voids – II: ellipticities and correlation function

Matteo Viel

Ellipticity



Voids Correlation Functions

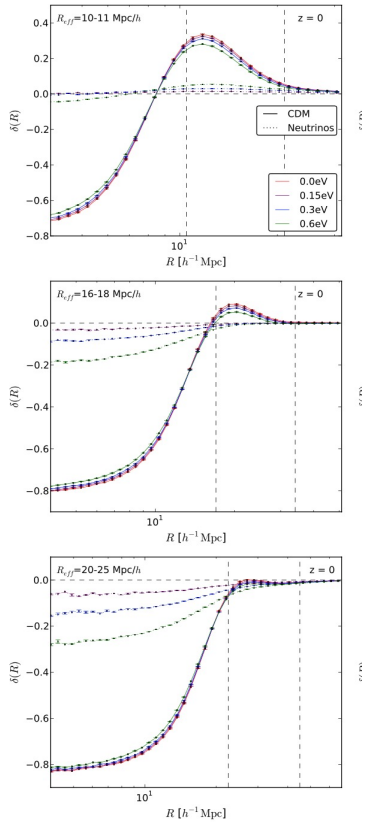


Exclusion effects

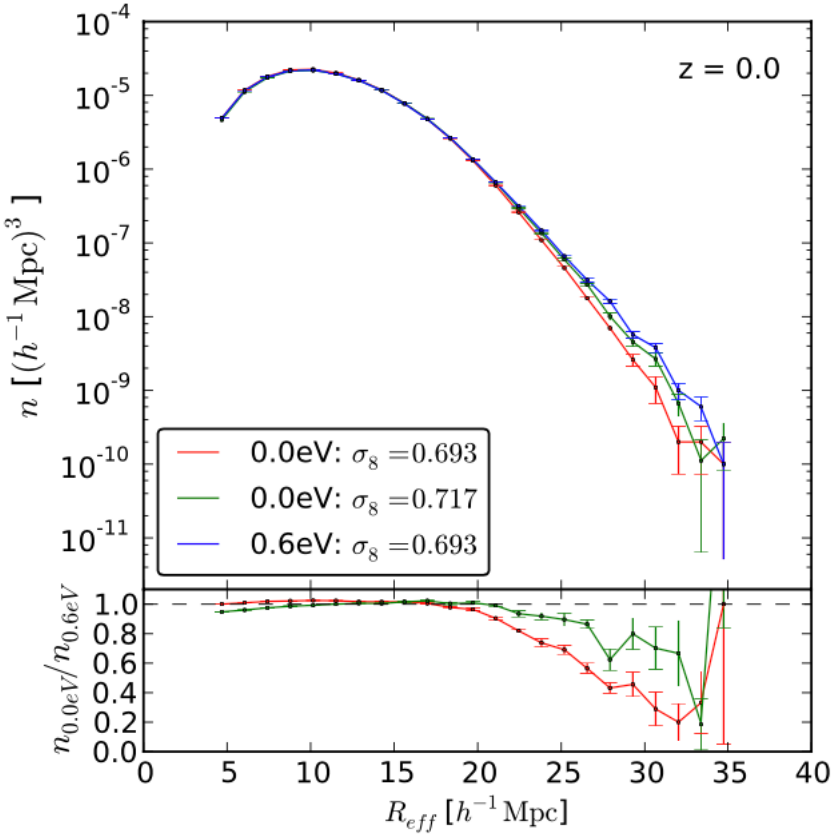
Voids – III: density profiles and degeneracies

Matteo Viel

Density profiles



Notice
The wall
For small voids

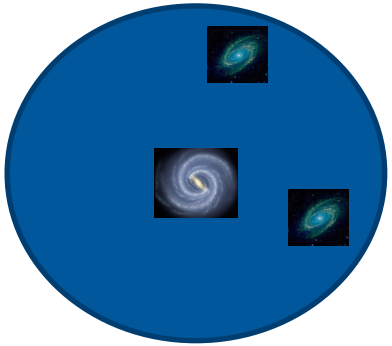


Voids – IV: populating with galaxies

Matteo Viel

So far: used the matter or halo distributions … but….real universe is different
We need galaxies --> HOD simple model

$$\langle N_c|M\rangle = \begin{cases} 1 & \text{if } M \geq M_{\rm min} \\ 0 & \text{if } M < M_{\rm min} \end{cases} \qquad \langle N_s|M\rangle = \begin{cases} (M/M_1)^\alpha & \text{if } M \geq M_{\rm min} \\ 0 & \text{if } M < M_{\rm min}\,. \end{cases}$$

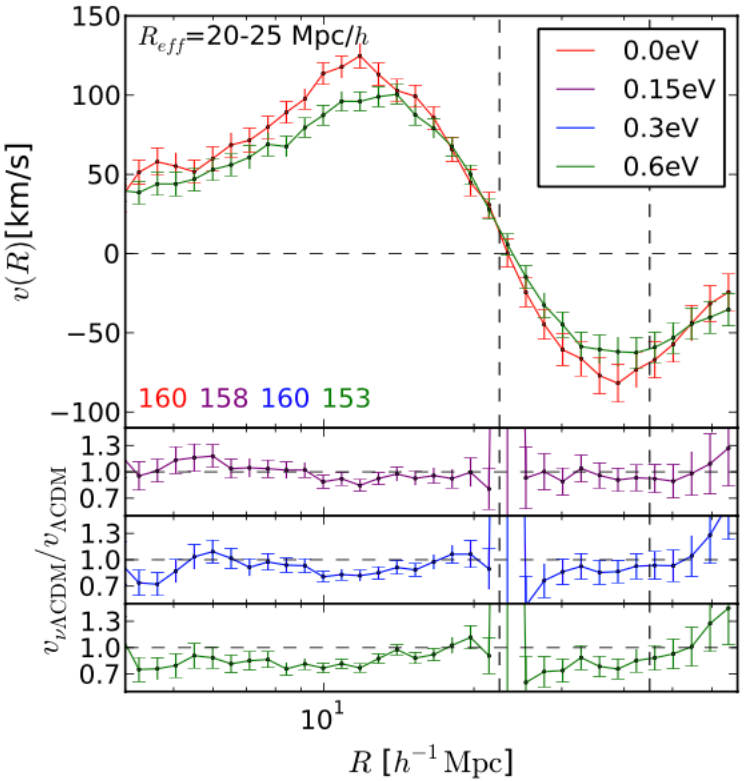


You can build simple HOD models
By asking to reproduce observed
Correlation functions or luminosity
properties of galaxies

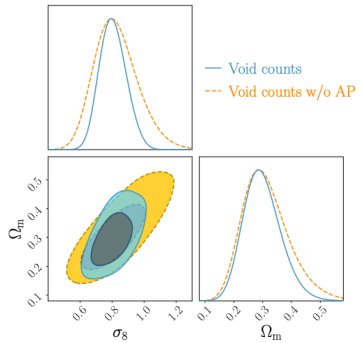
Once you build such a model you can then run your voidfinder on top of the galaxy distribution

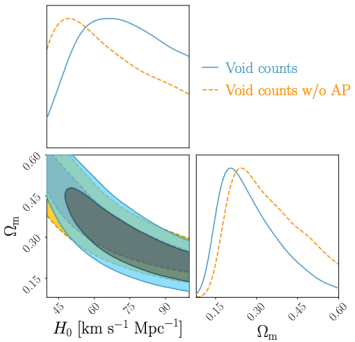
Voids – V: populating with galaxies

Matteo Viel



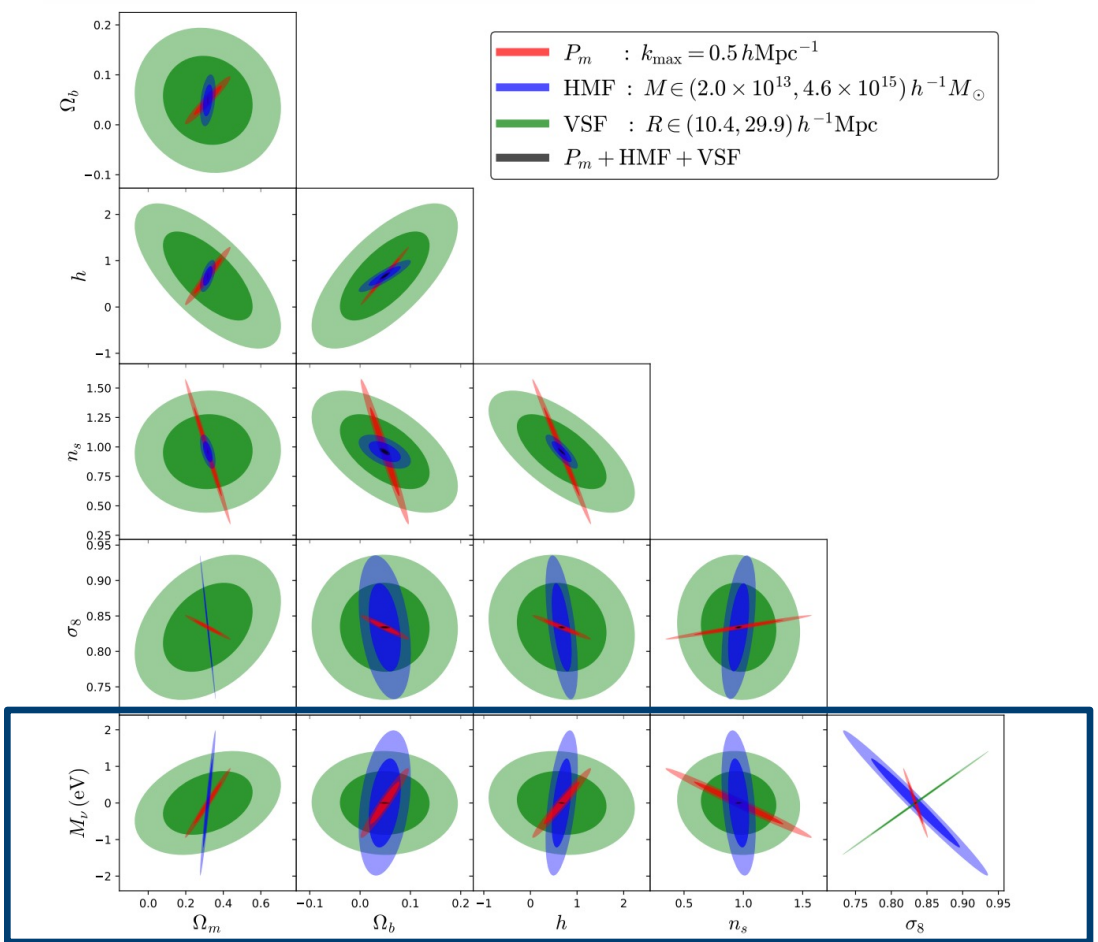
Voids on cosmology tensions Contarini et al.





Voids – VI: combining with other probes

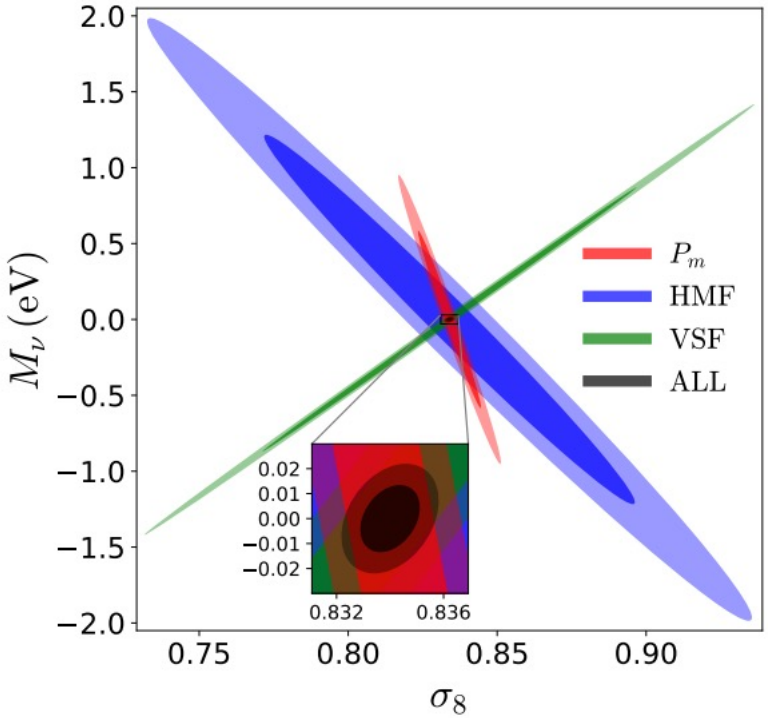
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Voids – VI: combining with other probes

Matteo Viel

Bayer+22 arXiv: 2102.05049



Marginalized Fisher Constraints						
Probe(s)	Ω_m	Ω_b	h	n_s	σ_8	M_ν(eV)
P_m	0.098	0.039	0.51	0.50	0.014	0.77
HMF	0.034	0.042	0.28	0.12	0.082	1.6
VSF	0.31	0.12	1.3	0.42	0.083	1.1
P_m + HMF	0.00077	0.0089	0.076	0.034	0.0016	0.061
P_m + VSF	0.016	0.011	0.12	0.074	0.0018	0.025
HMF + VSF	0.0063	0.037	0.23	0.10	0.0069	0.096
P_m + HMF + VSF (diag)	0.0015	0.0088	0.066	0.028	0.00061	0.031
P_m + HMF + VSF (auto)	0.0015	0.0086	0.071	0.033	0.0016	0.025
P_m + HMF + VSF (full)	0.00071	0.0084	0.064	0.025	0.0015	**0.018**
Multiplicative improvement	137	5	8	20	10	43

Without CMB priors to “fix” the large Scales

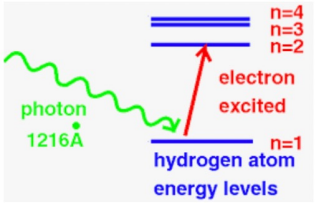
Volume 1 (Gpc/h)3, kmax=0.5 h/Mpc
From sims

But real surveys will have 100 more volume

21cm – atomic processes & hydro sims

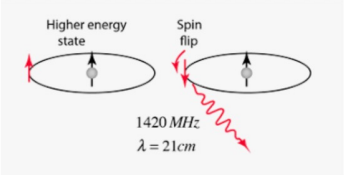
Matteo Viel

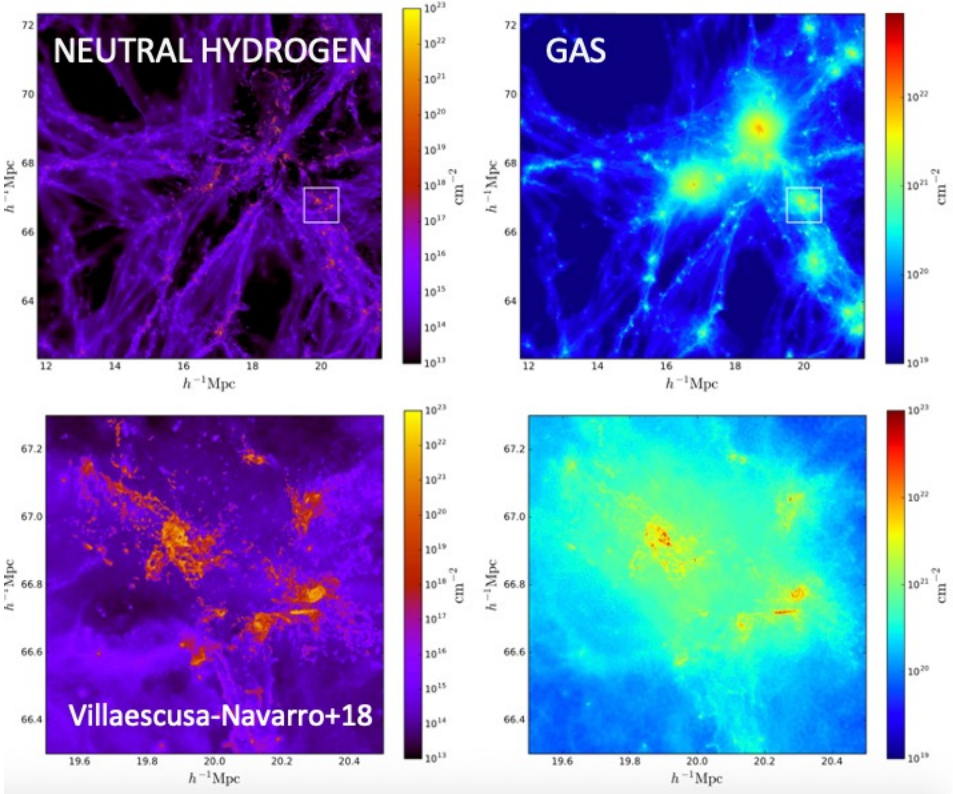
Atomic processes



Absorption - Lyman-alpha forest

Emission - 21cm intensity mapping





21cm – A simple model

Matteo Viel

Linear theory model:

$$P_{21\,\mathrm{cm}}(k,\,\mu,\,z) = \bar{T}_b(z)^2[(b_{\mathrm{H\,I}}(z) + f(z)\mu^2)^2 P_{\mathrm{m}}(k,\,z) + P_{\mathrm{SN}}(z)],$$

$$\bar{T}_b(z) = 189h\left(\frac{H_0(1+z)^2}{H(z)}\right)\Omega_{\mathrm{H\,I}}(z)\ \mathrm{mK},$$

$$\Omega_{\mathrm{H\,I}}(z) = \frac{1}{\rho_{\mathrm{c}}^0}\int_0^\infty n(M,\,z)M_{\mathrm{H\,I}}(M,\,z)dM,$$

$$b_{\mathrm{H\,I}}(z) = \frac{1}{\rho_{\mathrm{c}}^0\Omega_{\mathrm{H\,I}}(z)}\int_0^\infty n(M,\,z)b(M,\,z)M_{\mathrm{H\,I}}(M,\,z)dM,$$

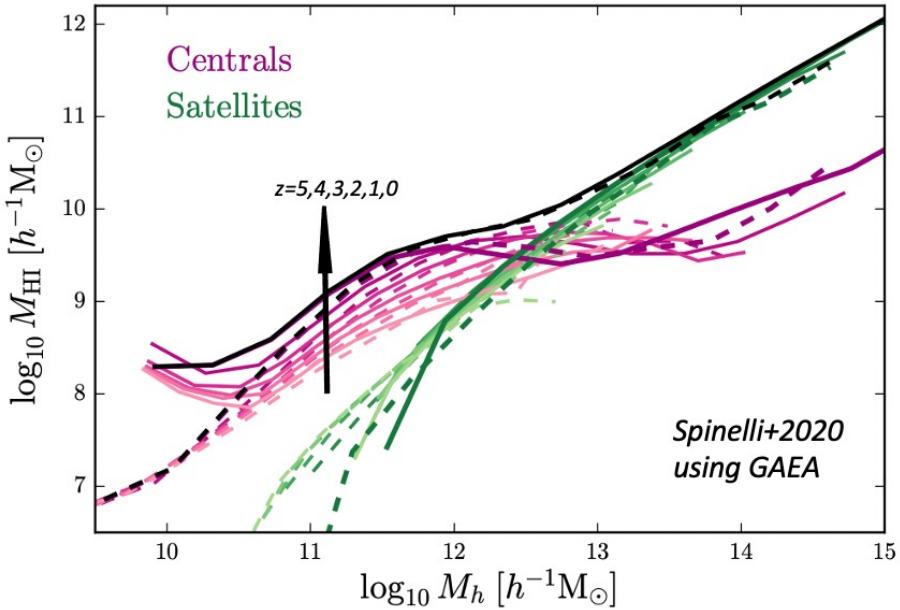
$$P_{\mathrm{SN}}(z) = \frac{1}{(\rho_{\mathrm{c}}^0\Omega_{\mathrm{H\,I}}(z))^2}\int_0^\infty n(M,\,z)M_{\mathrm{H\,I}}^2(M,\,z)dM,$$

$$M_{\mathrm{H\,I}}(M,\,z) = M_0\left(\frac{M}{M_{\mathrm{min}}}\right)^\alpha \exp(-(M_{\mathrm{min}}/M)^{0.35}).$$

M_{min} decreases with redshift
alpha increases with redshift

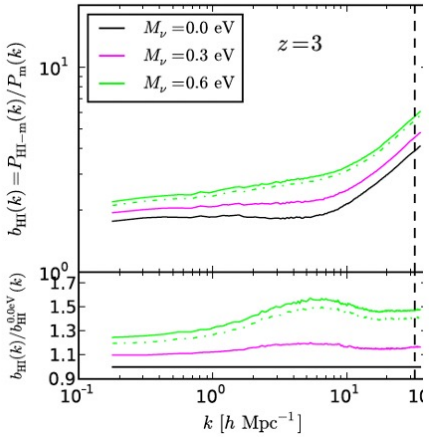
- degeneracy between b_{HI} and $Omega_{HI}$, which can be broken by using other probes (cross-corr.)
- Progress made mainly in the modelling and in determining the low-z HI bias (~0.8) from observations (Obuljen+18) - Pen+09, Switzer+13 (auto and cross to constrain Omega_HI x bias_HI), Anderson+18 (cross. with galaxies).
- **IM signal:** main ingredient is the function $M_{HI}(M_{halo})$ with its scatter.

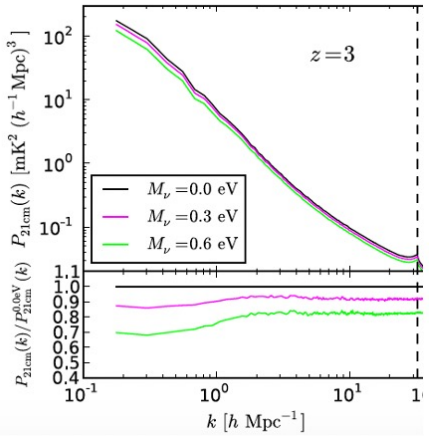
21cm – Cross correlations?



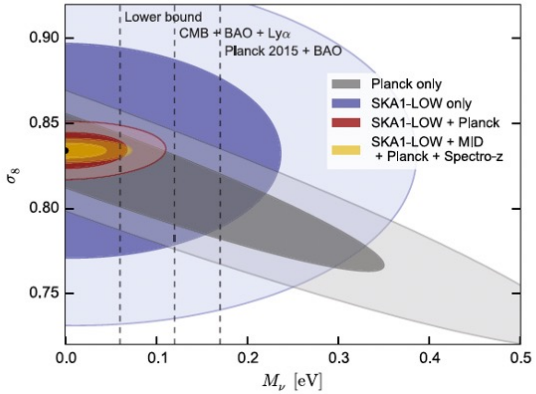
…further progress: interfacing this "small-scale" accurate and physical information with large scale methods for extensive mock productions e.g. PINOCCHIO LPT light-cone halos (Spinelli, Carucci+2021)

21cm and neutrinos





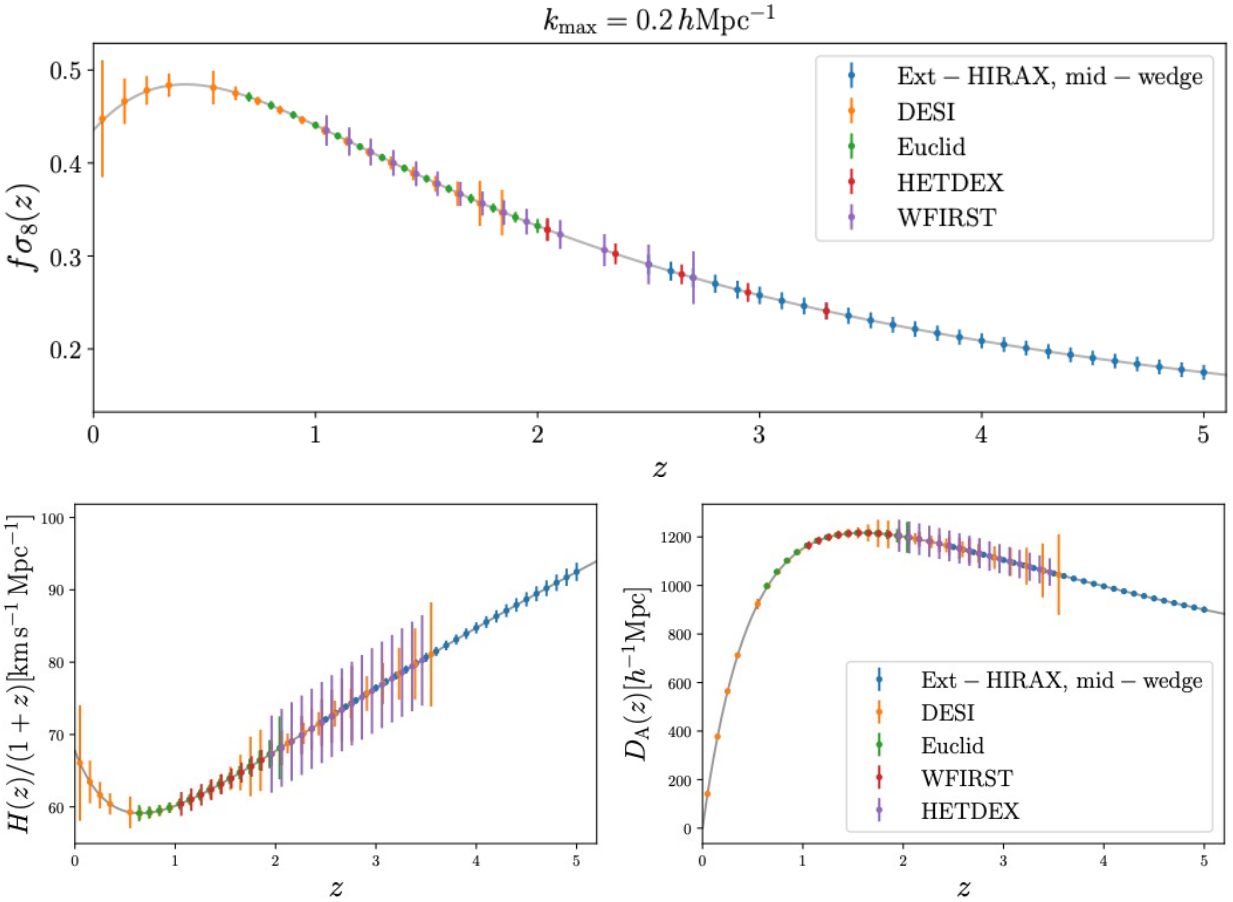
- Scale dependence bias also present in massive neutrino cosmologies.
- MHI(M) not affected by the presence of neutrinos.
- HI is more clustered in massive neutrino sims. (but OmegaHI lower) - because small mass haloes are suppressed i.e. impact on nHALO(M).
- IM alone would provide constraint of about sigma(M_nu) = 30 meV (not very constraining compared to other probes).
- Radiative transfer postprocessing important but does not impact much the limit above



Villaescusa-Navarro, MV, Bull, 2015

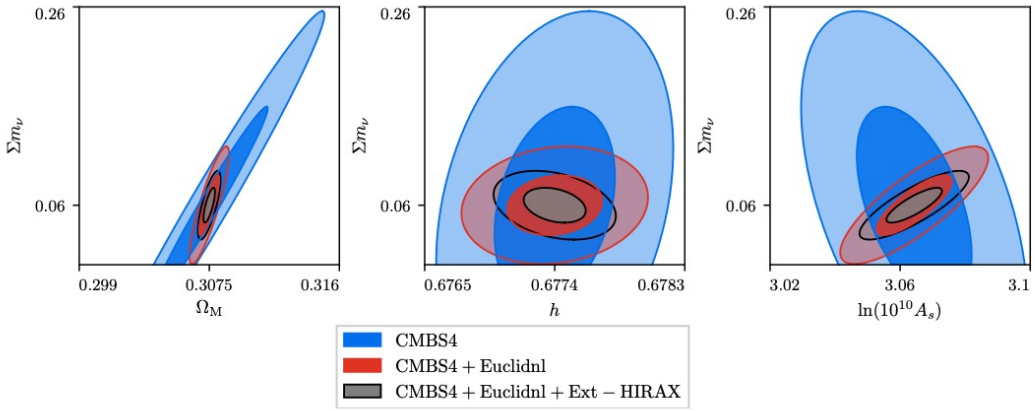
21cm and forecasts

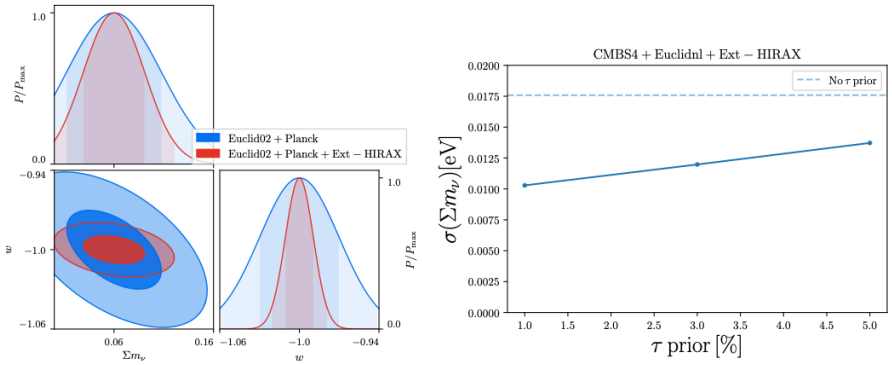
Obuljen+18



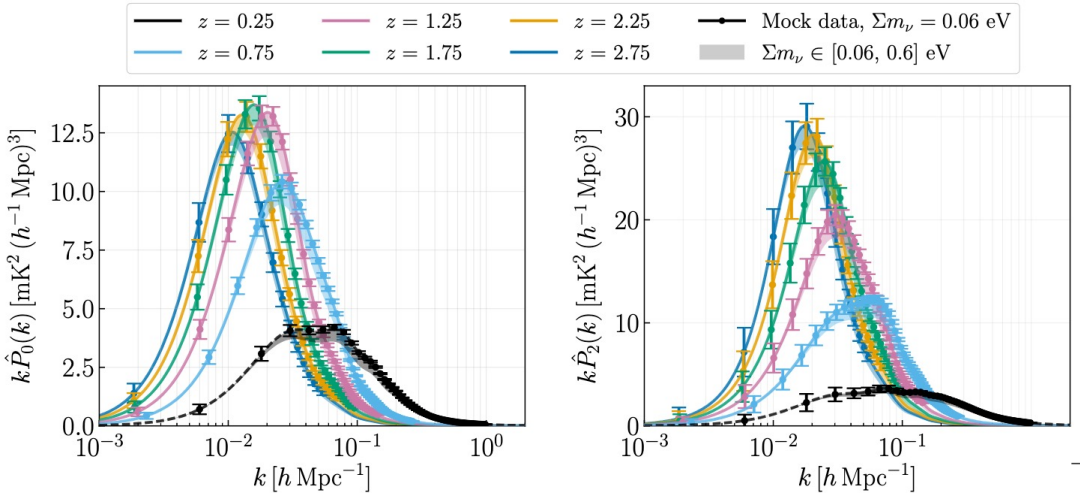
21cm and forecasts

Obuljen+18





21cm and forecasts

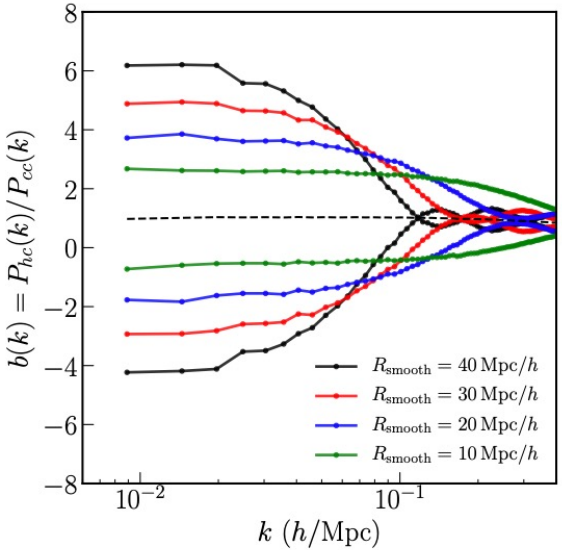


Berti, Viel, Spinelli 23 [to appear]

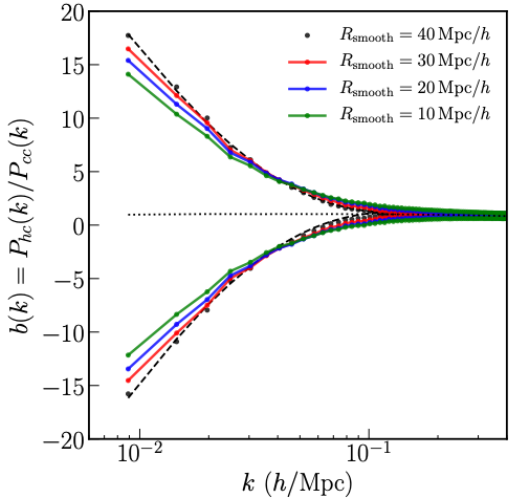
Likelihoods		$\Sigma m_\nu^{\mathrm{fid}} = 0.1$
$\hat{P}_0 + \hat{P}_2$	< 0.216	< 0.227
$+$ nuisances	< 0.478	< 0.535
$\hat{P}_0 + \hat{P}_2 +$ BAO	< 0.227	
$+$ nuisances	< 0.413	
Planck 2018	< 0.259	
$+ \hat{P}_0 + \hat{P}_2$	< 0.101	< 0.117
$+$ nuisances	< 0.129	< 0.127
Planck 2018 $+$ BAO	< 0.149	
$+ \hat{P}_0 + \hat{P}_2$	< 0.101	
$+$ nuisances	< 0.130	

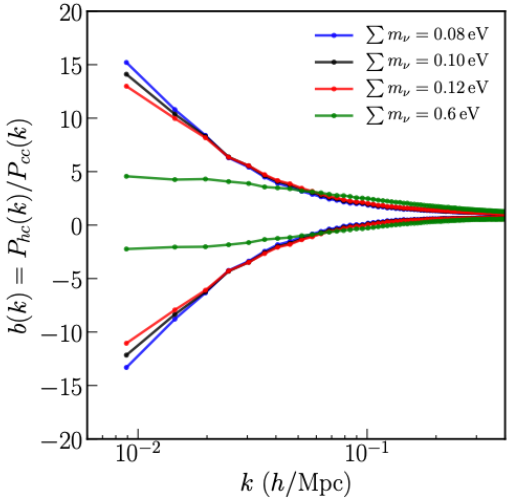
Environmental effects

Clustering of haloes in a massive Mn=0.1eV which are below and above the median CDM density



Clustering of haloes in a massive Mn=0.1eV which are below and above the median neutrino density



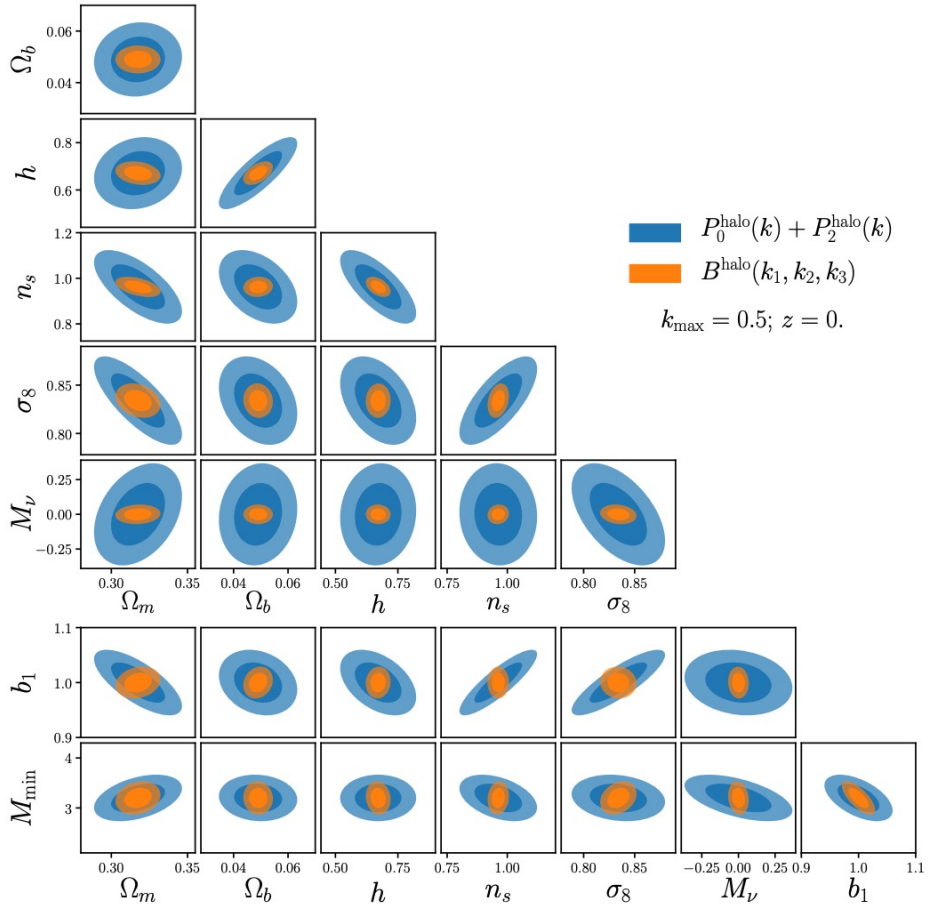


Strong scale dependence found → need a way to probe environment

From galaxies to halo to cdm…..

Banerjee, Castorina, Villaesusa-Navarro, Court, MV, 2019

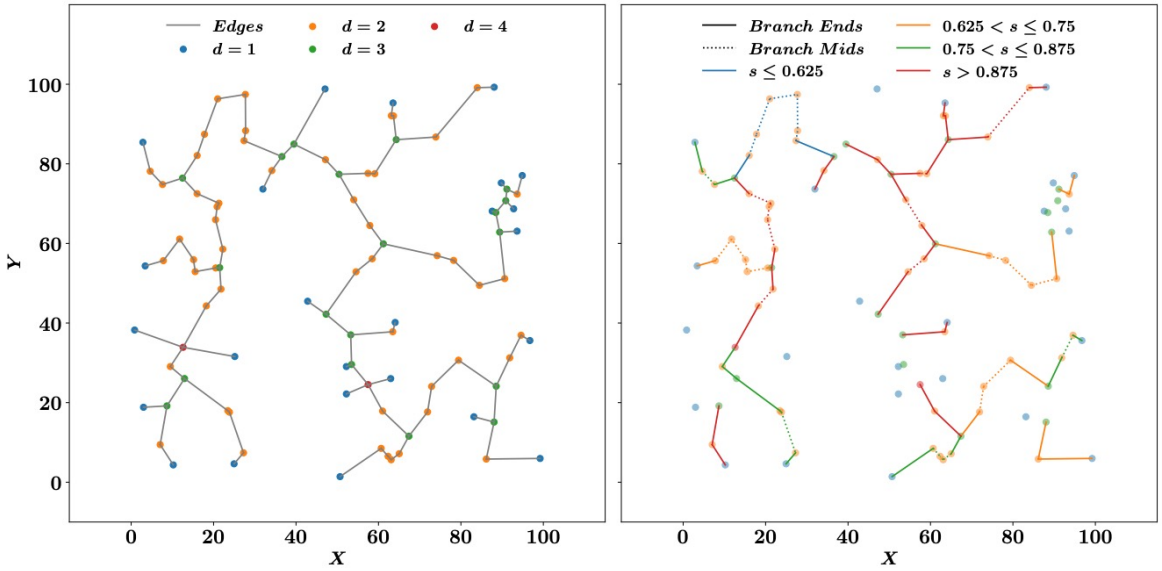
Bispectrum

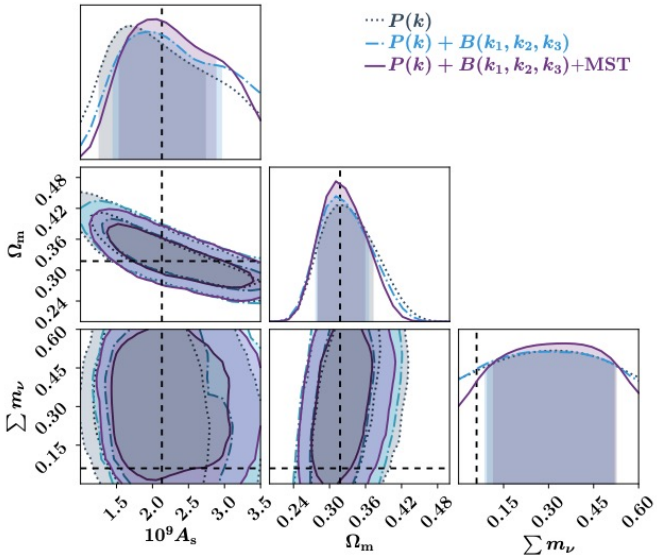


Hahn+19

Minimum Spanning Tree

Matteo Viel





Statistics which is extra-sensitive to topology of clustering

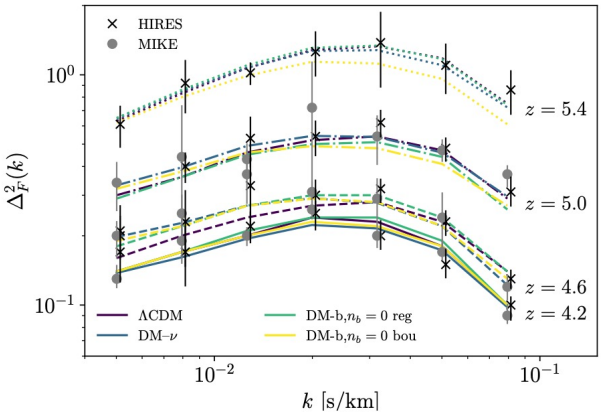
Naidoo+19
Naidoo+22

Summary

- Neutrino background detected from CMB through Neff
- In the structure formation epoch, neutrino perturbations not detected yet
- Effect is small but not incredibly small: whitin reach of planned and ongoing experiments (like Euclid)
- Neutrino free streaming is scale and z depdendent → should be tested/seen by different experiments subjected to different systematics/statistical errors
- Neutrino non-linearities are very interesting. Neutrino fluid reacts differently When considering different environments (galaxies, voids, etc.)
- Neutrino constraints from cosmology – so far a success story! [Could have been wrong.. But so far.. All predictions were correct]
- Suprises still possible: modification/new physics beyons standard models

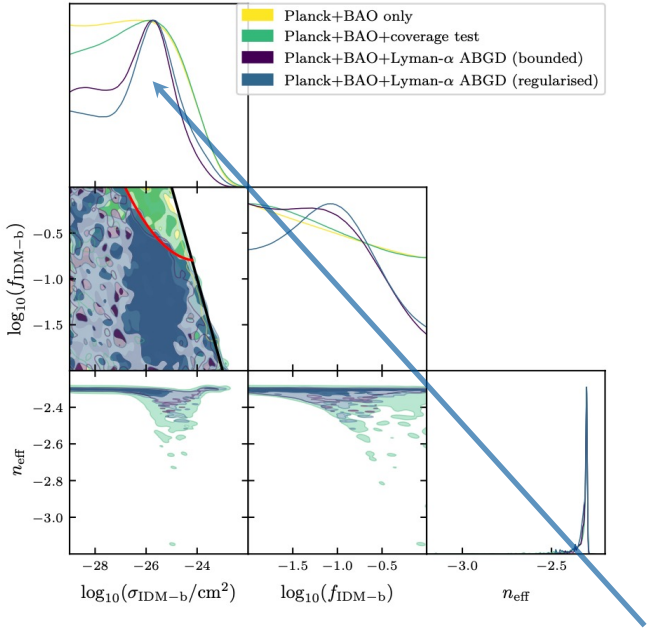
Dark Matter – baryon scattering n_b=0

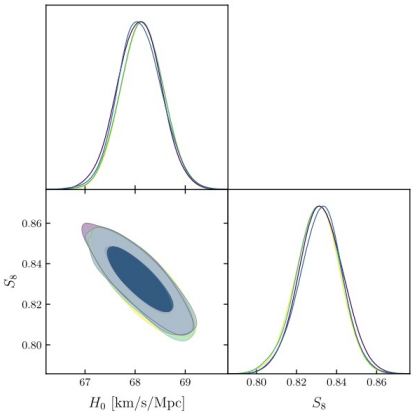
Matteo Viel



$$\sigma_{\rm T} = \sigma_{\rm DM-b} v^{n_b}$$

Hooper+22



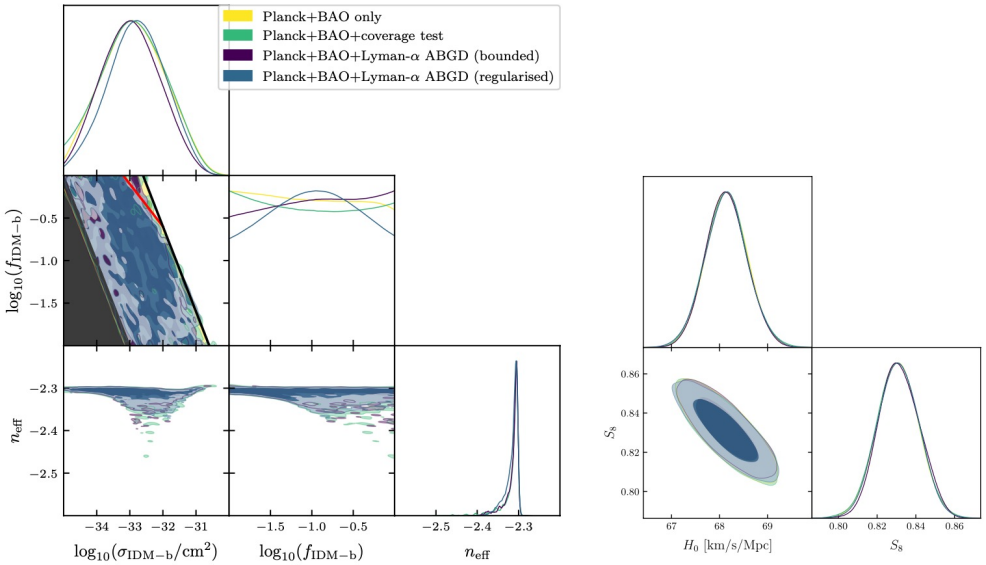


Momentum transfer cross-section
v: relative DM-b velocity
n_b=0 contact interactions, n_b=-2 in DM models with dipole moments

Hint of a detection
Slight overall tilt in the power spectrum

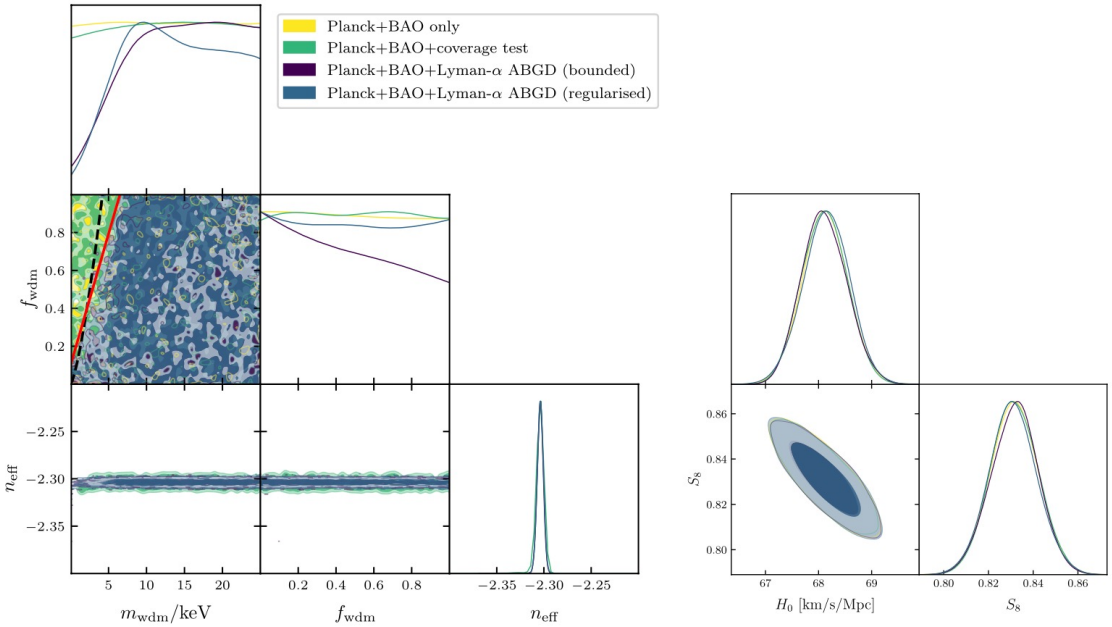
Dark Matter – baryon scattering n_b=-2

Matteo Viel



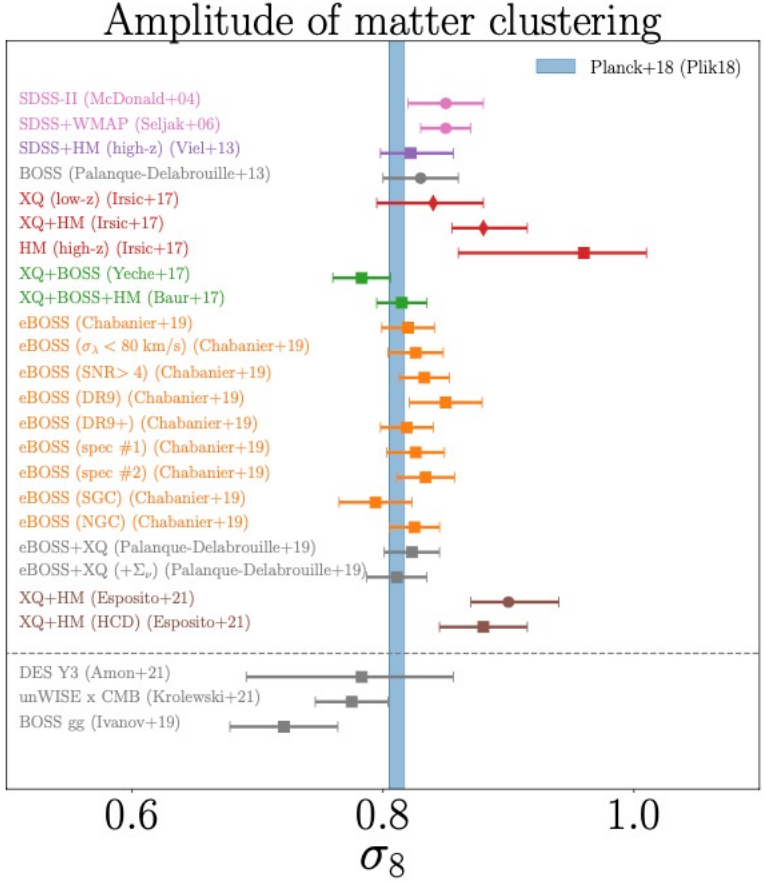
$$\text{Planck + BAO:}\quad \log_{10}(f_{\rm DM-b}) < -32.6 - \log_{10}(\sigma_{\rm DM-b}/{\rm cm}^2)\,,$$
$$\text{+ Lyman-}\alpha\text{:}\quad \log_{10}(f_{\rm DM-b}) < -16.6 - 0.5\log_{10}(\sigma_{\rm DM-b}/{\rm cm}^2)\,.$$

Cold+Warm Dark Matter

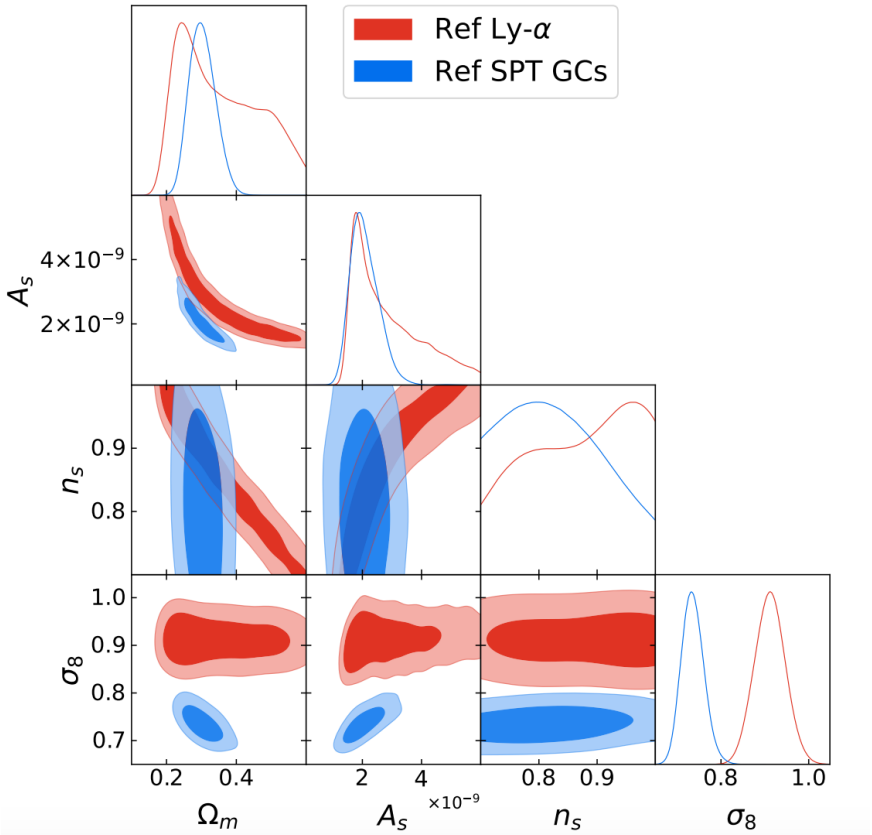


$$m_{\rm WDM} \gtrsim 7.2{\rm keV}(f_{\rm WDM} - 0.1)$$

σ_8 tension – I



σ_8 tension – II

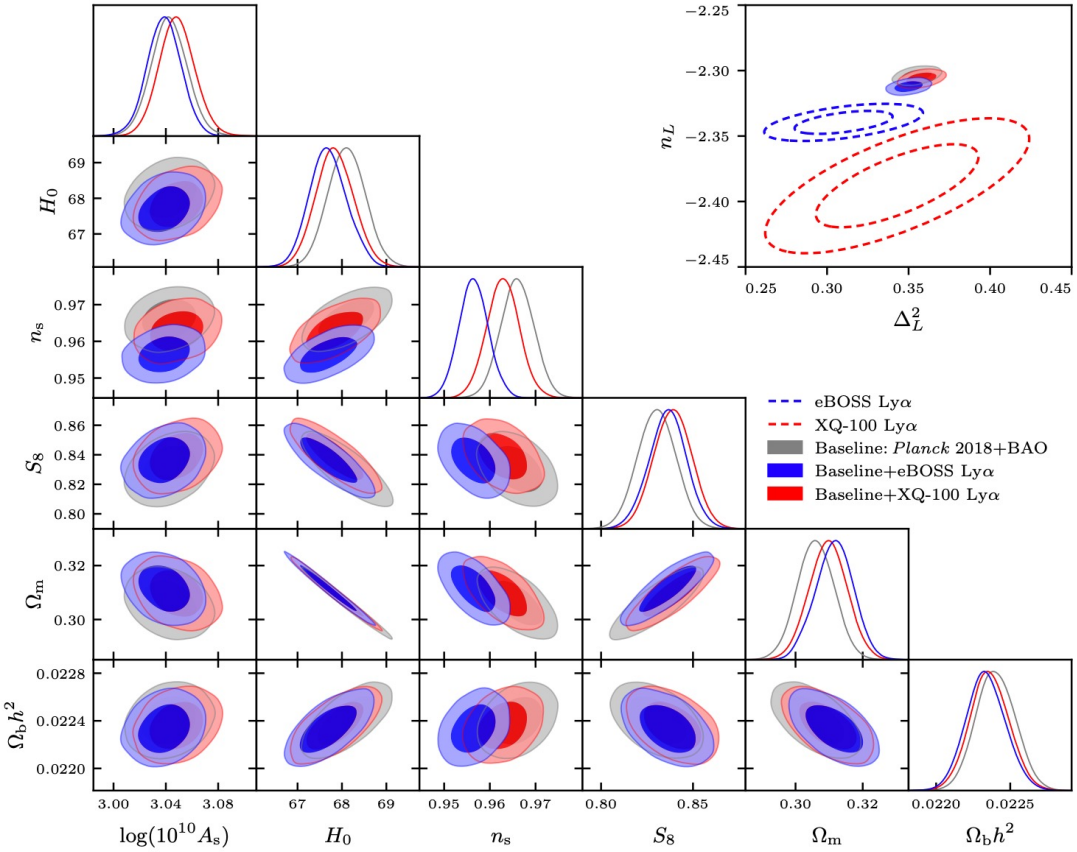


SPT GCs number counts
Lyman-alpha

Esposito, Irsic, Costanzi, Saro, Borgani, Viel 2022

Lya forest and LCDM

Goldstein+23



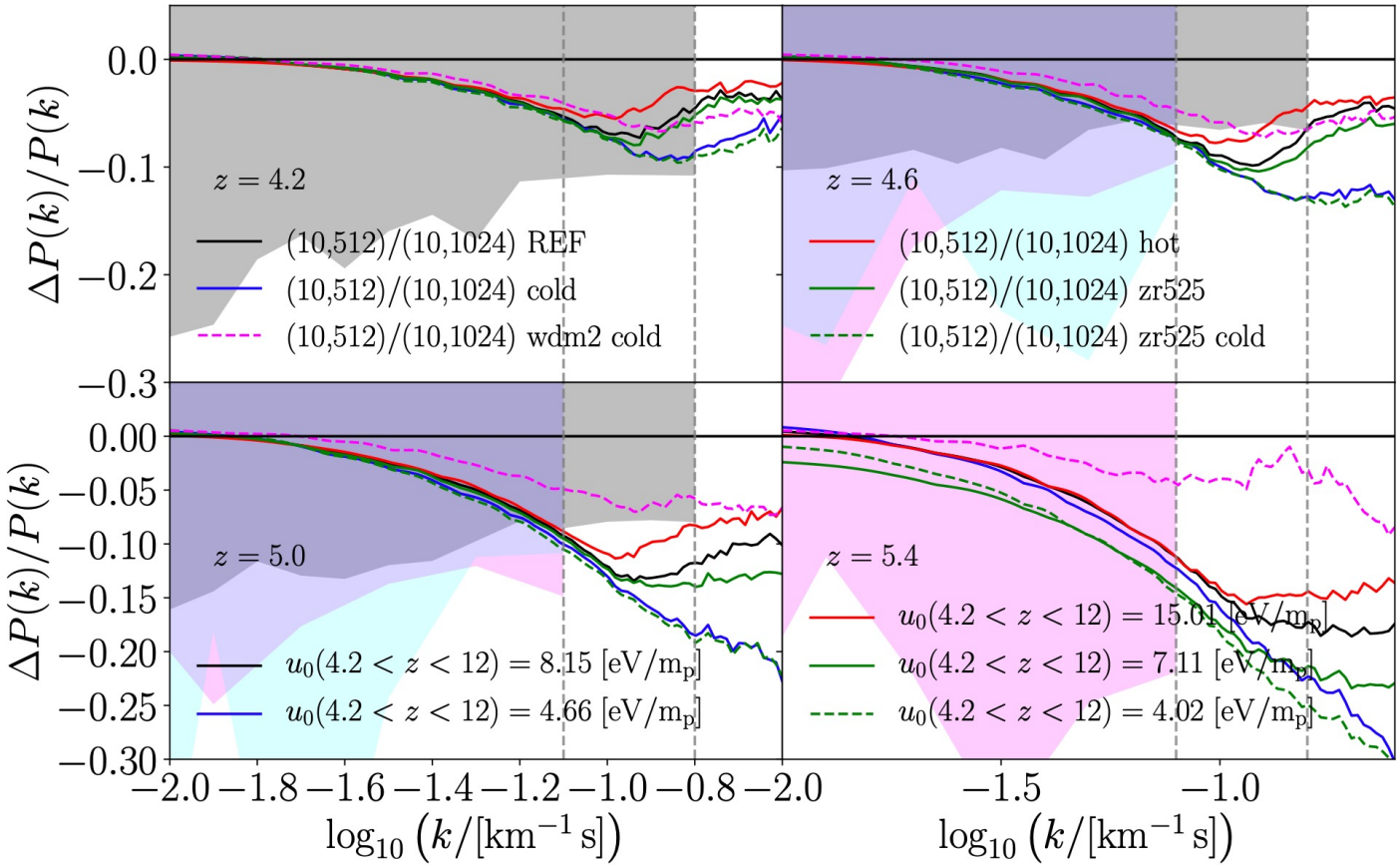
Discussion - Summary

Matteo Viel

- New data with new analysis: 5.72 keV 2s C.L. on WDM thermal mass
- Small scale regime of flux power is not easy to fit
  if you stop at k<0.1 s/km then 4 keV is a robust and conservative limit
- This includes new features: patchy reionization,
  resolution corrections depending on physical models.
  new unprecedented set of physical models for the simulations which
  fits several other statistics at once.
- This means: our results are prior driven!
- Pushing to small scales is double and hitting the regime > 6keV is likely
  to depend a lot on noise modelling….. But…. ESPRESSO, ANDES…
- New physical scale identified in 1D flux power (produced by pec.vel. Structure)
- Application to: dumping heating in the IGM → Dark Photon
- Application to: non standard DM-b and DM-DR interactions

Numerical convergence does depend on physics

Matteo Viel



Lya forest and EDE

Canonical Hubble-Tension-Resolving Early Dark Energy Cosmologies are Inconsistent with the Lyman-α Forest

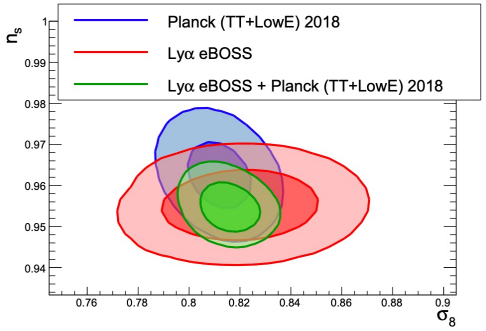
Samuel Goldstein [1,*] J. Colin Hill [1] Vid Iršič [2,3] and Blake D. Sherwin [4,2]

[1] *Department of Physics, Columbia University, New York, NY 10027, USA*
[2] *Kavli Institute for Cosmology, Madingley Road, Cambridge CB3 0HA, UK*
[3] *Cavendish Laboratory, Cambridge, 19 J. J. Thomson Ave, Cambridge CB3 0HE, UK*
[4] *DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK*

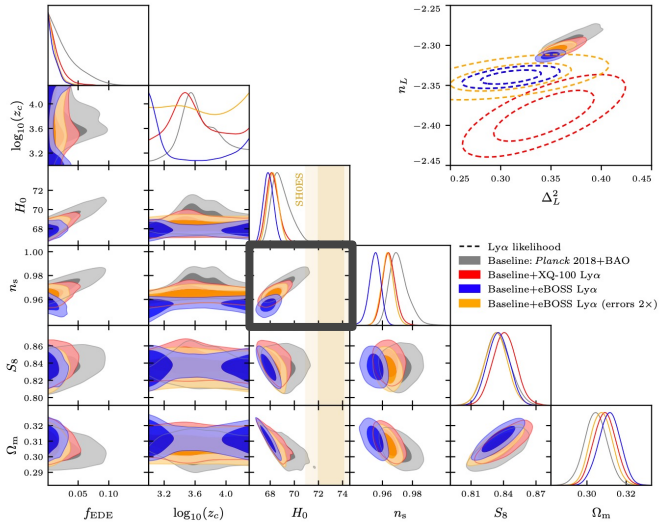
(Dated: March 2, 2023)

Current cosmological data exhibit discordance between indirect and some direct inferences of the present-day expansion rate, H_0. Early dark energy (EDE), which briefly increases the cosmic expansion rate prior to recombination, is a leading scenario for resolving this "Hubble tension" while preserving a good fit to cosmic microwave background (CMB) data. However, this comes at the cost of changes in parameters that affect structure formation in the late-time universe, including the spectral index of scalar perturbations, n_s. Here, we present the first constraints on axion-like EDE using data from the Lyman-α forest, i.e., absorption lines imprinted in background quasar spectra by neutral hydrogen gas along the line of sight. We consider two independent measurements of the one-dimensional Lyα forest flux power spectrum, from the Sloan Digital Sky Survey (SDSS eBOSS) and from the MIKE/HIRES and X-Shooter spectrographs. We combine these with a baseline dataset comprised of *Planck* CMB data and baryon acoustic oscillation (BAO) measurements. Combining the eBOSS Lyα data with the CMB and BAO dataset reduces the 95% upper bound on the maximum fractional contribution of EDE to the cosmic energy budget, $f_{\rm EDE}$, from 0.08 to 0.03 and constrains $H_0 = 67.9^{+0.4}_{-0.4}$ km/s/Mpc (68% confidence level), with maximum *a posteriori* value $H_0 = 67.9$ km/s/Mpc. Similar results are obtained for the MIKE/HIRES and X-Shooter Lyα data. Our Lyα-based EDE constraints yield H_0 values that are in $> 4\sigma$ tension with the SH0ES distance-ladder measurement and are driven by the preference of the Lyα forest data for n_s values lower than those required by EDE cosmologies that fit *Planck* CMB data. Taken at face value, the Lyα forest severely constrains canonical EDE models that could resolve the Hubble tension.

Chabanier+19 eBOSS analysis

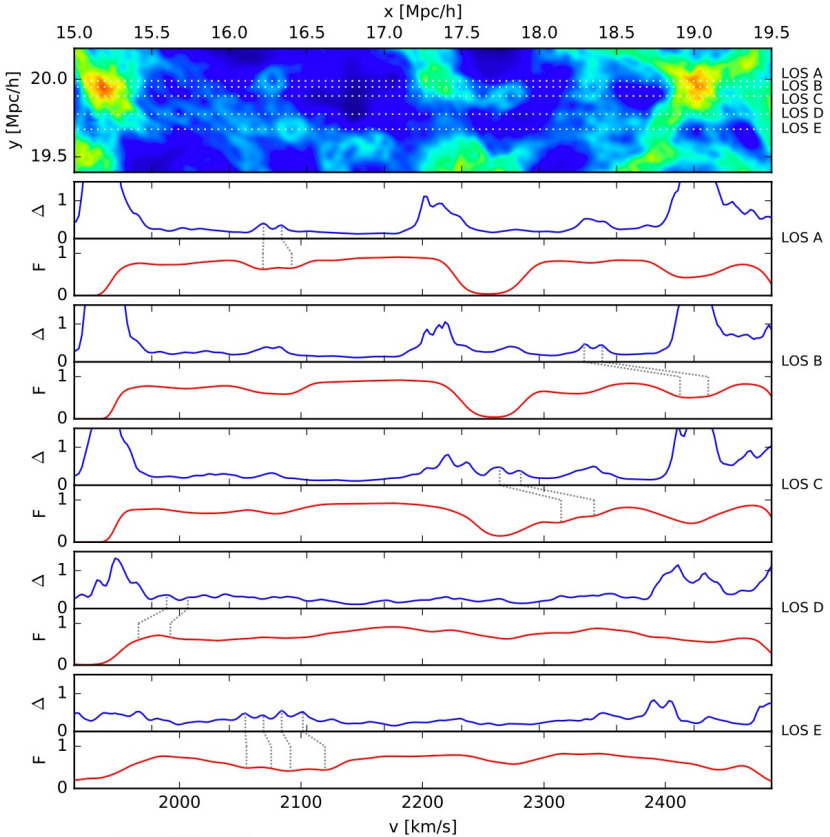


Assumes prior
H0=67.3 \pm 1 km/s/Mpc



New physical effects on the forest

Matteo Viel



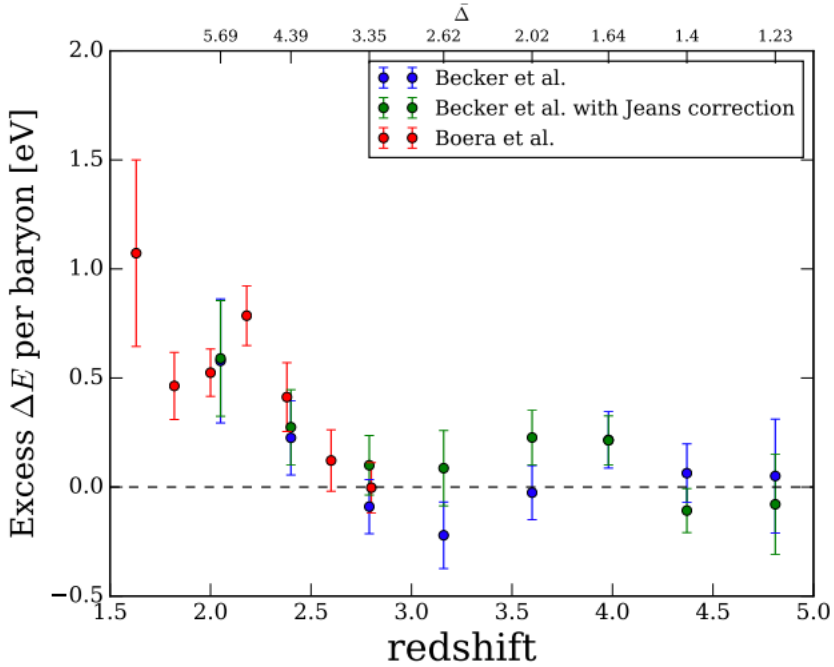
Pressure smoothing puffs up or evaporates small IGM structures such as filaments and small halos.

This often results in shell-like features in the IGM density field and can leave characteristic imprints in the Lyman-α forest, such as flat-bottom or double-dip absorption profiles.

Puchwein+23

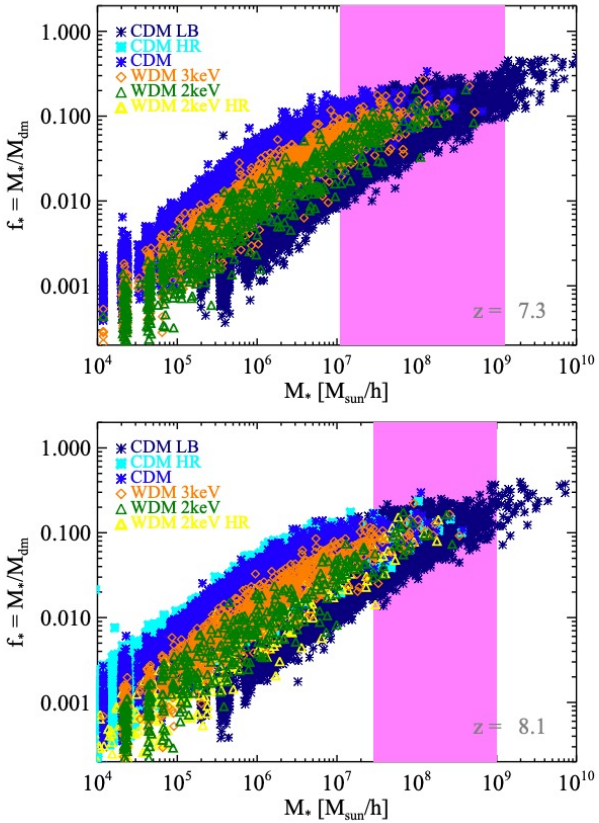
Extra heating in the IGM

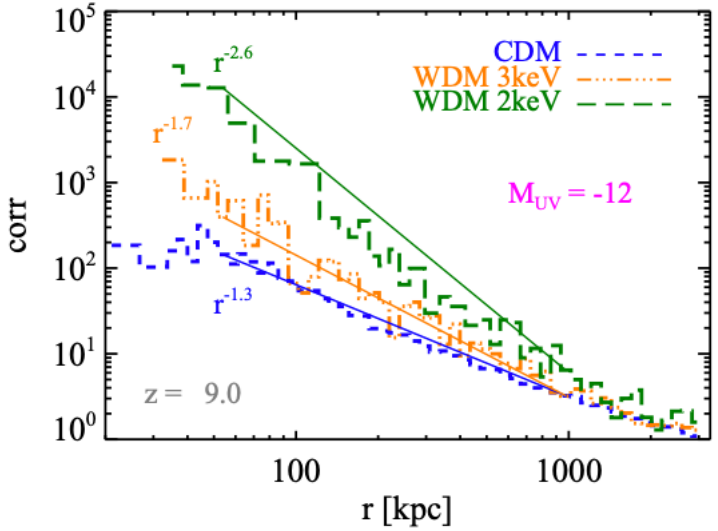
Matteo Viel



Upton-Sanderbaeck+15

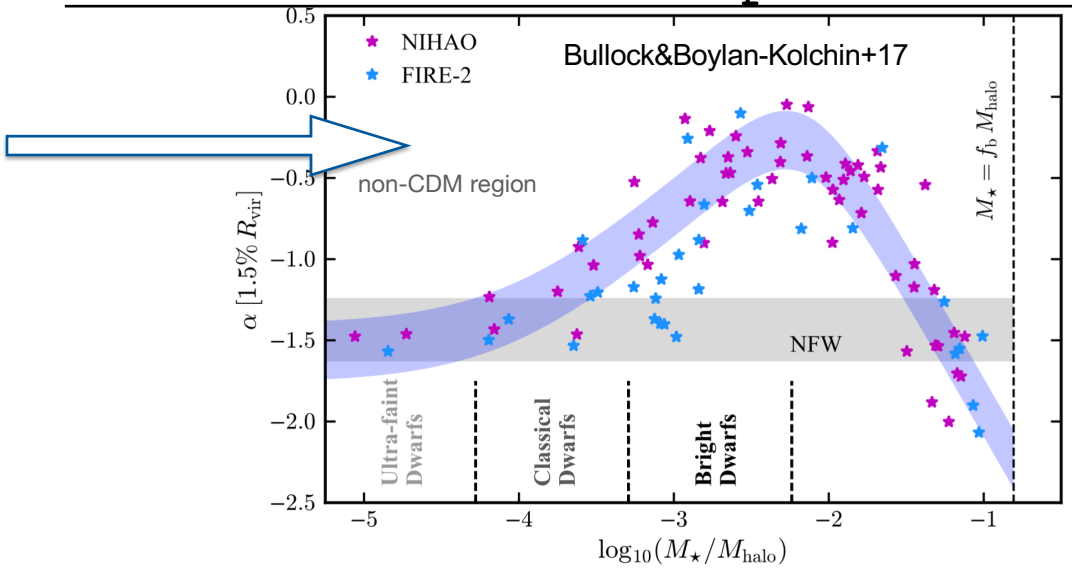
WDM and high-z galaxies





Maio & Viel 2023

ΛCDM model: core/cusps with feedback



Hydro simulation in LCDM with feedabck predict cored profile

for bright dwarfs 10^7-10^9 M⋆, and cuspy for classical (10^5-10^7 M⋆) and ultra-faint Dwarfs (10^2-10^5 M⋆)